# ESSAYS ON THE STRATEGIC USE OF ADVERTISING BY FIRMS

By

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The dissertation examines dynamic monopolistic and competitive strategic advertising policy as well as joint advertising and pricing policies. A theoretical model of a new consumer durable with no repeat purchase within the time horizon considered is developed. The dynamic demand, modeled in the monopolistic case as a linear function of the advertising is modified in the duopolistic case to include the effect of competitive advertising. In an extension of the model the dynamic demand is adapted to include price. The dynamic and static advertising and pricing policies in a duopolistic environment have also been investigated empirically. The estimation is carried out using a simultaneous equations model with nonlinear restrictions. The bi-monthly data used are for a nonseasonal household consumer product. The static competitive behavior of firms is empirically modelled using the conjectural variations approach. Dynamic behavior is captured by means of lagged Cournot reaction functions for price and advertising. An empirical estimation procedure for threshold effects of advertising expenditure is also suggested and implemented.

The research shows that a constant level of advertising is optimal for monopolists using advertising either as the sole strategic variable or in conjunction with price. In the duopoly situation a declining level of advertising is revealed to be optimal. The risk attitude of the firm is shown to determine its optimal advertising policy. The empirical investigation shows that the dynamic Cournot model better explains firm behavior in the market analyzed compared to the static models. The threshold effect of advertising is shown to operate at a positive level of change in advertising as opposed to simply a nonnegative one.

# CHAPTER 1
# INTRODUCTION

If we examine the costs of any firm, we can see that considerable amount of resources are devoted to advertising the firm's product to potential and present customers. Bass (1979) found that advertising expenditure varied between 30 and 400 percent of after-tax profits. Given these figures it is apparent that firms feel that advertising is an essential element of their marketing arsenal. The question is how much should they spend on advertising in order to maximize their profits. This question can only be answered after we understand the functions of firm advertising and how or under what circumstances consumers are affected by the intensity of advertising.

The four main functions of advertising that have been widely discussed in the economics and the marketing literature are, (a) providing information about goods to consumers (Telser 1964), (b) persuading customers to use the firm's product as opposed to that of its competitors, (c) erecting an entry barrier to other firms, and (d) complementing the other elements (e.g. price) of an overall marketing strategy of the firm. The importance of these functions is likely to vary across industries and econometric estimation is what can determine how important a specific function is for a particular product group.

The informational aspect of advertising is most obvious when a new product is being launched in the market. The consumers are faced with a whole spectrum of products on which to spend their money and will make their choice based on the products that are known to exist. Unless they are informed by means of advertising, consumers will not even consider buying a new product which has just entered the market. The medium of information advertising is often tailored to the stage of the product's life cycle. In the case of a new durable product whose initial prices are high and is targeted at high income consumers the advertisements of the product could appear in leisure magazines or special interest publications. Once the learning effect sets in and the costs begin to fall, informational advertising will begin to appear in mass media like television and daily newspapers.

Friedman (1983) points out that even firms manufacturing an established products need to remind customers of its existence, especially when the product is a specialized one. When a person's interests change either due to the adoption of a new hobby or change in the mode of living, information about products relevant to the new interest will be necessary. Friedman cites the example of photography as a hobby where special equipment and supplies exist which a novice photographer will not be aware of. In this case advertisements in photography magazines will be the prime source of information. It may be observed that the magazine 'Popular Photography' always seems to carry the very same product advertisements until a new generation of the same product is marketed by the company.

The informative aspect of advertisements can be taken one step further when an advertisement provides technical details about the product to the consumer. Using the photography example once again, the consumer knows that telephoto lenses are needed for long distance shots but may not be aware of what focal length lens to buy. A company sales catalog provides the consumer with the necessary information and may even make him aware that by buying a zoom lens he can have an entire menu of focal lengths at his disposal. Thus advertising can serve the basic and useful function of disseminating information about product attributes to potential consumers. Alba and Hutchinson (1990) argue that the extent to which advertising can provide useful information depends on the level of expertise a consumer has using that type of product. In the context of our photography example, a more experienced amateur photographer can better understand the advantages of using specialized equipment listed in a sales catalog.

As most consumers know, firms often advertise far in excess of what is required to convey basic information to consumers. Nelson (1974) points out that this apparent excessive advertising can still convey information to buyers if the product is an 'experience' good. An experience good is defined as one whose quality cannot be determined without actual purchase. The apparently wasteful excess advertising can signal the unknown quality of experience goods. Nelson reasons that higher quality brands will obtain more repeat purchases, ceteris paribus, than low quality brands. Thus the sellers of the high quality products will advertise more to get consumers to try their product since the present value of such purchases is greater. This, according to Nelson, causes the better firms to advertise more in equilibrium given the fact that consumers react to advertising. The level of advertising for experience goods is thus positively correlated with quality regardless of the veracity of individual advertisements since the amount of information is what signals quality to the consumer, not the claims such advertisements make.

The persuasion aspect of advertising can operate at two different levels. Firstly, it can persuade the consumer to change tastes relative to other products which compete for a place in the consumption bundle. In terms of standard indifference curve theory what we are saying is that the indifference map can be made to change through advertisements if we make the assumption that the consumer has a well-defined indifference map at any instant in time.

There is obviously an intertemporal aspect to this induced change in taste -- it may occur only gradually over a period of time. This is specially true for goods for which a consumer develops or acquires a taste like some exotic variety of food. This type of persuasion is relevant when a monopolist is operating in the market.

After advertising persuades consumers away from competing product categories and expands the total market size as a whole, the second level of persuasion comes into play. With a mature nondifferentiated product, advertising is one of the main nonprice devices used to obtain a competitive edge over rival firms. Given the fact the rival firms behave in the same way, there will be similar advertising by them in response. This competitive response aspect of advertising can have long term and short term components. A competitor can of course respond by altering other factors like price or quality but Aaker and Carman (1982) remark this is unlikely to happen because quality change is a slow process and price competition is usually avoided in oligopolies. In general, when there is no more room for primary demand to increase, competitive or noncooperative behavior in oligopoly will cause all firms in the market to counter advertising with advertising.

The persuasion aspect of advertising operates through the creation of brand loyalty. One measure of brand loyalty is the proportion of an individual's total purchase within a given product category devoted to the most frequently purchased brand or set of brands. Ornstein (1984) remarks that the assertion that advertising creates brand loyalty has imbedded in it an unspoken theory of consumer behavior, complete with a description of the psychology of consumer buying and the process of consumer learning. This psychological basis of brand loyalty is usually not addressed in the economics literature where a general belief prevails that buyers tend to gravitate towards an established brand name product in preference to a less advertised brand. The reasoning behind this belief is that consumers perceive that established brands imply less risk with regard to the quality of the product. Comanor and Wilson (1974) offer a theory for the reasoning stated above. They postulate that the problem of consumer choice between brands in a given product class is largely a problem of decision making under incomplete information and a risk averse attitude on the part of the consumers.

The primary sources of information according to Comanor and Wilson are experience in product use and advertising. The consumers can be persuaded to buy a product either by means of a discounted price or greater information conveyed through advertising. Thus in their model, the persuasion function of advertising works through the information function of advertising since it persuades by providing more information. Better known brands can command a higher price because of the higher level of information available to consumers about it. Brand loyalty is said to be created not by emotional appeal of advertising, but by the relative differences in information across brands obtained through experience and advertising.

The role of advertising expenditure as a barrier to entry has been extensively examined. Two different ways in which advertising can form a barrier have been identified. First, entry barriers may be formed if there exists economies of scale in advertising. Second, advertising can create durable preferences for an incumbent firm's products which are difficult to recreate thus making for a product differentiation advantage barrier. Waterson (1984) identifies two intersecting strands to the argument on advertising scale economies corresponding to technical and pecuniary economies. Technical economies of scale occur when sales increase in greater proportion than the increase in advertising. After the advertising messages have reached the most likely buyers the effect of additional advertising is bound to be much less. Thus it seems likely that such technical economies of scale are somewhat limited. Regarding the pecuniary economies of scale, it can be argued that cost per message falls as advertising expenditure increases. Advertising scale economies combined with possible production scale economies can be effective in preventing or delaying entry, even when the established industry earns above normal profits. Advertising can also be a source of economies of scope if complementary products like computers and printers are advertised together.

If we consider advertising in the dynamic context where advertising has a lasting effect then we get to the second type of entry barrier effect of advertising. In the absence of price competition, the share of industry advertising expenditure is one of the major determinants of industry output share. The problem that entrants face is that goodwill due to advertising takes some time to build up, which means that they will get an output share far below their long run equilibrium share of output in the initial periods. The higher the level of advertising by the incumbents, the greater are the losses that they will suffer, even accounting for higher profits in later periods. The important thing here is that the barrier to entry is caused by profits being delayed, not because of heavy capital requirements. Schamalensee (1974) pointed out that for durable brand preferences to create a barrier to entry, increasing returns to advertising must exist i.e. it should be a sunk cost.

Established brand names can often constitute a barrier to entry through the limiting of access to distribution channels. Any new brand of consumer product that comes into the market has to face extremely stiff competition for shelf space in supermarkets. One way of getting around this distribution channel access problem is by joining forces with store chains where product manufacturers sell their products under what are known as private or store labels. Good examples are store brand cosmetics marketed by K-Mart and Walmart chains. Shelf space for these products is obviously assured in addition to being priced substantially below the standard brand name products. If we imagine a brand name to generics continuum then private labels will fall somewhere in the middle since their prices and name recognition are at an intermediate level between the two. The manufacturers of private label products often compete against well known brand names in related products. An example of this is the private label soups that are manufactured by Heinz and which compete with the near monopolist in that market, Campbell Soups.

We can argue that advertising by incumbents can remove barriers for a late entrant rather than creating them. In terms of the product life cycle theory, the incumbent initially has to incur advertising costs in order to create demand by informing the consumers about the product. The doubting majority must be convinced that this is an useful product. If an entrant comes in just when the incumbent has taken the product to the latter part of the growth period or early part of the maturity period, it can save advertising expenses. At the mature stage two conflicting forces come into play. The brand loyalty factor will become important and the entrant will have to incur advertising expenses in order to establish itself in the minds of the consumers. The degree of standardization of the product on the other hand will facilitate entry since no product differentiation barrier will be in effect.

The advertising decision is an integral part of the marketing mix decided on by the firm and as such advertising could affect the effectiveness of some of other marketing variables. The effect of advertising on the price elasticity of demand is an example of such effects. If advertising decreases price elasticity then high advertising should accompany high prices. If the reverse is true, a low price may be selected even if there is a large expenditure on advertising. Existing empirical evidence substantiates either hypothesis regarding the effect of advertising on price sensitivity (to be discussed in the next chapter). Advertising expenditure can be used as a substitute for promotional expenditure since both of them direct the attention of the consumer to the product. Advertising decisions by the producer often have to be coordinated with the advertising done by distributors. The local advertising expenditure by the dealers has to be coordinated with national advertising campaign of the manufacturer.

The empirical research involving advertising has gone in different directions in the economics and marketing literature. The main concern of the economics literature has been the relationship between concentration and the advertising-sales ratio. In the earlier studies like Cable (1972) a straightforward OLS regression of advertising to sales revenue on Herfindahl concentration index and the number of brands was done. The conclusions of these studies were later thought to be suspect because of a possible simultaneous relationship between concentration and advertising-sales ratios. The basis for the existence of such a relationship, suggested by Ornstein (1984), is that larger firms could have higher price-cost margins because of greater efficiency which implies higher possible levels of advertising on their part. But since larger firms are important in concentrated industries, there is a causal link going from the concentration ratio to the advertising-sales ratio. Strickland and Weiss (1976) correct for such simultaneity bias but end up estimating a simultaneous model which does not pass the sufficient condition for identification.

The focus of the empirical research involving advertising in the field of marketing has been primarily been concerned with examining the relation between brand loyalty and advertising. If advertising creates brand loyalty, empirical studies should demonstrate a significant relationship between advertising expenditures and measures of brand loyalty. Ornstein (1984) remarks that greater market share stability in high-advertising industries and low rates of new-brand introduction in brand-loyal industries can also be expected to be related to advertising expenditure. The reason for this is that there should be less switching across brands by consumers in brand-loyal markets, and the expected returns from new-brand introduction should decrease this brand loyalty. From the examination of studies by Telser (1962), Lambin (1976), Reekie (1974) among others, there does not emerge any clear evidence supporting the hypothesis that advertising creates brand loyalty. The results of Buzzell and Nourse (1967), Telser (1975), Porter (1978) and Henning and Mann (1978) indicate that market shares are more unstable in advertising-intensive industries, and new brand introductions are highest in advertising-intensive industries.

Given the fact that firms usually operate in an oligopolistic environment, it is important to account for competitive interaction in empirical research regarding advertising. Studies such as Gazmi and Vuong (1988) test for different kinds of noncooperative behavior in advertising expenditure using restrictions on the sales response functions. The first order conditions and sales response functions are estimated together as a simultaneous system. The conjectural variations approach can also be used to assess the effect of competitive reaction.

One major controversy about advertising in the economics literature is with regard to its welfare implications. If we could make the assumption that consumers had perfect information about whatever goods they buy then the information role of advertising disappears, forcing us to conclude that all such expenditure is a social waste. Since consumer information is not perfect, advertising serves a useful social function by providing product information thus allowing the consumer to make a more informed choice. However the problem arises when it serves purposes other than that of information -- namely, persuasion or predation.

Predatory advertising is said to occur when the general reduction of advertising would leave the demand functions unaltered. This means that the purpose of advertising is to effect a transfer of profits between firms within the same industry. From the industry point of view cooperative advertising would be far more suitable here since it could serve to attract customers away from other products competing for the consumer dollars. Evidence of joint advertising is not common among manufacturers of the same product but can be observed at the dealer level. For example, within a geographical area Chevy car dealers often advertise cooperatively on television. However, even interindustry predation is socially useless since once again it implies transfer of profit between firms. The problem is that it is difficult to distinguish between the information and the predation components of advertising. This interwoven dual nature of advertising prevents us from saying unequivocally that advertising is a social waste.

In the following chapter we will look at some of the existing literature on advertising in both economics and marketing fields which address the issues outlined above. In the third and the fourth chapters a theoretical model of a new consumer durable with no repeat purchase within the considered time horizon is examined. The dynamic demand is initially modelled solely as a linear function of advertising in the monopolistic case and is then modified to include the effect of competitive advertising in the duopolistic case. The dynamic demand is also extended to include price and uncertainty. A differential game framework is used to analyze the duopolistic case. The specific problems investigated within this framework are the following:

a. The advertising policies of a monopolist and a duopolist when dynamic demand depends only on advertising.

b. The pricing and advertising policies of a monopolist and a duopolist when dynamic demand is a function of both price and advertising.

c. The behavior of the myopic monopolist when faced by an uncertain time of entry by a second firm.

d. The advertising policy of the monopolist when it believes that it can deter entry by means of an advertising barrier.
e. The advertising policies of a monopolist and a duopolist when demand is stochastic in nature.
f. The advertising policy of the monopolist faced with two types of uncertainty, namely, stochastic dynamic demand as well as uncertain timing of entry.

In the fifth chapter we empirically investigate the dynamic and static advertising and pricing policies for a nonseasonal household consumer product. The static competitive behavior of the firms is empirically modelled using the conjectural variations approach. Dynamic behavior is captured by means of lagged Cournot reaction functions for price and advertising. The specific types of firm behavior tested are the following:

a. Price and advertising matching behavior with unitary conjectural variations i.e., firms are hypothesized to exactly match changes in price and advertising by the competitor.
b. Cournot-Nash behavior with zero conjectural variations i.e., firms do not react to changes made by a rival in its current period price and advertising levels.
c. Dynamic Cournot behavior where firms set current period price and advertising based on the previous period price and advertising levels of both firms.

We also estimate the threshold level of advertising expenditure at which it has a pronounced effect on sales, using the two phase regression method. In the sixth and concluding chapter we discuss the implications of the results and indicate avenues for future research.

# CHAPTER 2
# REVIEW OF LITERATURE

Advertising is probably the most common form of non-price competition undertaken by firms. This has led to a great deal of work concerning the different aspects of advertising discussed in the previous chapter, in both the economics and marketing disciplines. In this chapter we will review some of the recent noteworthy contributions pertaining to the functions and effects of advertising.

Examination of the informational role of advertising tells us that product differentiation must exist if advertising by individual firms is to convey information. In the absence of product differentiation, the whole industry could advertise cooperatively to compete as a product group. This appears to be the case in product categories such as fruits where differentiation is difficult. The products must differ with regard to one or more attributes. Information about these attributes can then be transmitted via advertising. This product differentiation can occur along any dimension, one of which is location. The Grossman and Shapiro (1984) model deals with this situation where advertising is incorporated into a location model of product differentiation where each firm's product is represented as a point in a circle of unit length. Consumers are assumed to be distributed around the circle with a constant number of consumers per unit length. The demand curve facing each firm is a function of the prices and advertising intensities of other firms. This demand curve is derived from the underlying preferences of consumers across brands, and the technology by which consumers obtain information through advertisements by firms.

Grossman and Shapiro show that within the confines of the circular product differentiation model, advertising serves only to inform and the resulting advertising levels in large-group oligopolistic equilibrium are above the socially optimal level. A second result they obtained was that reduction in the cost of advertising per exposure increases the competitiveness of the market measured in terms of demand elasticities and causes prices to fall. The main drawback of this model is its circular spatial orientation which limits the generality of the results. A general model of product differentiation where products have multiple differentiable attributes is required to validate the results. Also, the consumer should not be limited to advertisements alone for information. Word of mouth and search behavior are alternative information sources for the consumer which can be modeled.

Nelson (1974) argued that apparently wasteful advertising expenditure could turn out to be useful if market mechanisms exist which produce a positive relationship between product quality and advertising expenditure. This idea of advertising acting as a signal of quality was modeled formally by Schamlensee (1978). He considers an experience goods market with multiple firms and consumers where the unit cost of producing a good of given quality is the same across firms. The production cost is assumed to rise with increase in quality. The incentive to produce high quality goods lies in the fact that these goods have higher conditional repeat-purchase probabilities. The central result obtained by Schamlensee is that low quality brands will make higher profits when there is increasing return to scale for advertising. This implies that it is possible for low quality brands to have an incentive to advertise more. This result was criticized by Kihlstrom and Riordan (1984) as being rationally inconsistent.

The Kihlstrom and Riordan model assumes that firms are price takers who influence their selling price through advertising. In the first variant of their model, they assume that consumers' experience with the product becomes common knowledge. Advertising only serves to influence the price the initial customers are willing to pay. This model shows that advertising can still be an effective signal of quality if high quality entails higher fixed cost than low quality firms keeping the marginal costs the same. If the marginal cost does go up, no advertising equilibria are shown to exist. In the second version of their model

Kihlstrom and Riordan assume that if all high quality firms except one advertise, the nonadvertising firms will not be able to establish a reputation for high quality. This model more closely follows Nelson (1974) in that it is advertising which leads to repeat sales and not word of mouth between consumers. Advertising equilibria are possible in this model even if marginal costs are higher for high quality firms as compared to low quality ones. The limitation of both variants of the model is that firms do not get to choose prices. In equilibrium prices are correlated with quality but are not used to infer quality. This defect was remedied by Milgrom and Roberts (1986) and Rogerson (1988) where both price and advertising are decision variables for firms and both can potentially signal quality.

Milgrom and Roberts show that for a new product, about which consumers do have much quality information both prices and advertising could be simultaneously used as signals of quality in equilibrium. If high quality firms priced or advertised at the full information level, it would render the signals useless since low quality firms can match these levels. Consumers would end up ignoring these signals since both types of firms would be pricing and advertising at the same levels. Rogerson (1988) shows that because price does signal quality allowing professionals to advertise increases consumer welfare. The Milgrom and Roberts model would become considerably more complicated if an established brand were considered because consumers are bound to have other sources of information including knowledge disseminated by previous buyers.

Rogerson (1987) shows that for an established brand, levels of brand name investment (such as advertising) increases with the increase in the elasticity of demand with respect to such investment. The dissipation of excess profits occurs through this process. This dissipation would occur through entry of more firms into the market if the elasticity of demand with respect to brand name investment were low. Bagwell and Ramey (1988) develop a model where price and advertising are used to signal an incumbent's costs in a scenario where a potential entrant is attempting to infer its costs. Their model shows that in such a case price is distorted downward regardless of whether advertising is demand-enhancing or not. Limit pricing can thus take place even when advertising is used to signal costs.

Kaserman and Mayo (1985) present a case where advertising is used to urge consumers to reduce consumption of residential electricity. The study pertains to the post oil-embargo period of the seventies when the rising prices of oil prompted conservation measures. The authors empirically investigate the effectiveness of such informational energy conservation oriented advertising and find that the advertising elasticity of residential electricity is very low.

Given that the persuasive aspect of advertising is what firms are concerned with most of the time, the issue of how sales respond to advertising is the main issue for advertising researchers. Little (1979) spells out the basic controversies and areas of research in his comprehensive article reviewing the models of aggregate advertising. After examining the available empirical evidence, Little concluded that models of aggregate advertising should demonstrate the following phenomena:

a. Sales respond dynamically to changes in advertising in the same direction and often do so at different rates.

b. In the long-run (steady state) sales response to advertising will have either the concave or an S-shape with positive sales at zero advertising.

c. Advertising by competitor firms affects the sales of firms.

d. The dollar effectiveness of advertising can change with the quality and media of advertising.

e. Sales may go up with advertising but may fall off if advertising is fixed at the higher level.

A great deal of the current literature is focused on the first three phenomena listed above. In particular, the competitive aspect of advertising has received a great deal of attention. The basic idea was that sales is a positive function of its own advertising and a negative function of the competitor's advertising. Game theory is used to find the equilibrium levels of advertising expenditure of each firm. The Cournot-Nash equilibrium is commonly used for this purpose. Stahl (1989) has used the symmetric Nash equilibrium concept to show that when both price and advertising are decision variables for firms, at equilibrium price approaches marginal cost as advertising costs decrease and tends toward the monopoly price level as they increase. The welfare effects in the model depend on the type of advertising technology adopted. If advertising costs are sufficiently convex, then entry of sellers makes lower prices more likely, with net social benefit. If advertising costs are linear, then entry of sellers makes higher prices more likely and results in negative social benefit.

When a new product is launched, the type of campaign that will be used is an important decision that firms have to make. At a general level they have to decide whether to go for a pulsing (strong bursts of advertising spread over the planning period) or an even advertising policy. Sasieni (1971) showed that when sales response decreases with increases in advertising the optimal strategy is to reach a desired level of sales and maintain advertising at a fixed level. A pulsing policy can never be optimal given the decreasing returns to scale of advertising. Sasieni also characterized a chattering policy where the firm alternates instantaneously between a zero and some positive level of advertising. He shows that this theoretical policy can substitute for the increasing returns to scale (convex) portion of the advertising response curve to elicit greater response. This chattering policy effectively linearizes the convex response portion and hence does better than operating along the curve. Mahajan and Muller (1986) model a S-shaped advertising function where they show that a chattering policy is better than an even advertising policy and a pulse type policy. Sasieni (1989) shows that sales response settles down to a steady cycle in the long run and the average response varies inversely with the length of the cycle.

Empirically it has been observed that firms seldom maintain the same level of advertising expenditure in every planning period. Dynamic models address the problem of determining the optimal rate of advertising expenditure over time so as to maximize the present value of a firm's net profit streams over a finite or infinite horizon. Sethi (1977) and Feichtinger et al. (1989) survey some of the major contributions in this area including dynamic game-theoretic models. Optimal advertising levels are obtained in these models by solving the optimal control problem where sales is the state variable and advertising expenditure is the control variable. For such models closed form solutions are often difficult to obtain and numerical methods of solutions have to be resorted to. This problem becomes more acute when one is dealing with an oligopolistic model. Tractability of these models is also adversely affected if a second control variable (say price) is added on. As a consequence of these complications results are derived from the optimal time path of advertising, i.e. its rate of change over time instead of the optimal level at any point in time.

Teng and Thompson (1983) developed a model of advertising competition where the production costs are subject to a learning curve. The effect of price on sales response is neglected in this model while advertising affects market share according to the Bass (1969) diffusion model. Since closed form solutions are not possible for the differential game that results in the oligopoly case, numerical simulation is used. This model demonstrates ways in which complex situations can be attacked using appropriate numerical algorithms. Dockner and Jorgenson (1988) model the innovation diffusion in a monopoly situation in the same framework as Thompson and Teng. The main analytical results they obtain are that in a monopoly situation where advertising is the only decision variable, optimal advertising should start low and increase or start high and then decrease. Decreasing advertising is optimal in the presence of positive diffusion effects and increasing advertising is optimal with negative diffusion effects.

Dynamic models incorporating more than one decision variable (i.e. two marketing mix variables) suffer from tractability problems. Interactions between the two variables causes problems for the derivation of closed form solutions. Thompson and Teng (1984) consider the case where both price and advertising are decision variables. For the monopoly case they show that a higher value of the production learning coefficient or a low price elasticity implies a higher level of optimal advertising. Three different pricing rules are suggested and tested numerically for maximization of profits. Numerical simulations are used in the oligopoly case due to the mathematical intractability of the resulting differential game. These simulation exercises indicate that profit and advertising rise with advertising effectiveness and the learning coefficient while the optimal price set by a price leader decreases over time. No clear relationship between price and advertising dynamic policy is derived.

Feichtinger et al (1988) consider the optimal price and advertising policies for a convenience good retailer where consumers choose a store based on price image and advertising. Price image refers to the extent the convenience store is thought to be a low price store by the consumers. In such a model it appears that the image of a convenience store can be improved mainly by pricing strategy, with advertising kicking in only after the price image reaches a certain level. However if the price image reaches a high level and then goes down to an equilibrium level, heavier advertising is advocated. The managerial implication of this result is that initially such a store would do better if it used the pricing strategy and reserved advertising only to reinforce its pricing image once it perceives a decline. Kalish (1985) models the dynamic innovation diffusion process which includes price and advertising and takes into account the uncertainty with regard to the product's experience attributes. The monopoly case is considered and the consumers are assumed to be risk averse in the sense that they are willing to pay more as information filters down from the early adopters. He finds that advertising decreases over time when profits are not discounted and decreasing returns to scale in advertising exists. Horsky and Simon (1983) obtained a similar result for a more restricted form of the advertising response function.

The effect of advertising itself can be dynamic apart from the problem of determining a dynamically optimal policy. This characteristic of advertising stems from the fact that often there is a carryover effect of advertising where advertising in the current period affects sales in the current and future periods. This was first modeled in the well known Nerlove and Arrow (1962) study using a goodwill term which summarizes the effects of current and past advertising outlays on demand. In the Nerlove-Arrow model, advertising in each period contributes to the stock of goodwill which continuously leaks away. This treatment of goodwill as an investment in a durable good was extended to the oligopolistic case by Fershtman (1987). He derives the optimal advertising policy for an individual firm proves the existence of an optimal level of goodwill for every market share. At equilibrium the firm's goodwill and advertising expenditure tend to decrease as the number of firms increases. Fershtman also shows that in the symmetric case the firms' goodwill and advertising vary inversely with the discount rate, the depreciation rate of goodwill and production cost.

The majority of the models discussed above have been developed for a deterministic environment. It is possible that adding uncertainty to the analysis will alter or modify some of the results. Raman (1990) considers uncertainty in the context of a advertising goodwill model. The optimality of the ratio rule of advertising (firms set advertising budget as a percentage of current or expected sales) under stochastic evolution of goodwill and sales is examined. Raman finds that under the condition that the probability distribution of goodwill and sales is lognormal the ratio rule is the optimal advertising policy under dynamic conditions. Conversely, when the distributions are not lognormal the optimal policy is nonlinear in nature.

Horsky and Mate (1988) developed a competitive diffusion model where the diffusion effect is uncertain. The competition between two rival firms is formulated as a two-player, nonzero sum, Markov game where the market evolves from state to state as a Markov process. It is shown that the optimal advertising policy for the two brands is to start with high advertising budgets and reduce them over time. The firm which lags behind in terms of market share will be the one to advertise more aggressively since its word of mouth effect is going to be less. A first-mover advantage which increases with the impact of the diffusion effects is shown to exist. This advantage can be countered by a late entrant who comes in with a superior product with a strong word of mouth from the early adopters.

Once an optimal advertising policy is determined, firms need to decide on the actual dollar figure and allocation of advertising across markets. Friedman (1958) is one of the few studies which systematically tackle these problems in a competitive environment. The Nash equilibrium concept is used to determine the allocation of advertising dollars across territories. Mixed strategy solutions are obtained in the case when both firms have a fixed advertising budget and sales go to the firm spending the most in advertising, all other things being equal. The limitation of the model is that a firm needs to know the competitor's total advertising budget in order to calculate its optimal allocation of advertising funds. This model is static in nature and hence not very useful for allocation purposes in a dynamic environment. A dynamic formulation incorporating uncertainty with respect to the competitor's advertising budget size should be of interest.

Nguyen (1987) incorporated uncertainty in a multibrand competitive environment to assess its effect on sales-advertising decisions as well as firm's advertising allocation decision. The competitive environment in the model consisted of both interfirm and intrafirm (brands of same firm) competition. The theoretical model indicates that, unlike in the single brand firm case, the effect of the firm's risk attitude and random sales environment is fairly complex. The single-brand result that the firm's advertising decreases as sales become more uncertain does not hold here. The model was estimated using cigarette industry data where this result was substantiated. It was found that within a given cigarette company, advertising levels for other brands within the company and the industry advertising level may decrease or increase sales level of a brand. The nature of this interaction can vary both from brand to brand and from company to company.

The entry prevention role of advertising has been widely debated in the economics literature. The basic argument made is that heavy advertising by an incumbent firm can create brand loyalty which proves to be a barrier for any entrant choosing to enter the market. Schmalensee (1974) considered a general distributed lag model of the dynamic demand functions facing established firms and potential entrants in a situation in which advertising is the only competitive weapon. The distributed lag structure of the demand implies that a firm's sales respond only gradually to changes in its determinants (advertising alone here) so an entrant would need to incur penetration cost. Schmalensee shows that only if capital markets have serious imperfections and entrants lack sufficient collateral can the market position of an established firm deter entry. In the absence of high learning costs an entrant will produce as efficiently as an incumbent and this would not offer any scope to the incumbent to deter entry. A high degree of demand asymmetry is necessary for it to be optimal for the incumbent firm to erect a barrier to entry with high advertising.

When advertising is modeled as a goodwill forming expenditure, it may be expected that an incumbent would increase advertising to build up goodwill so that entry will become less profitable. Schmalensee (1983) showed exactly the reverse, i.e. the incumbent firm will reduce its advertising to prevent entry instead of increasing it. The model developed is one of two stage competition where firms choose advertising levels sequentially or simultaneously in the first stage and play Cournot-Nash in the second stage. A fixed cost of advertising and a homogenous good is assumed. The intuition behind the result stated above is that a large incumbent facing a small entrant will have a flatter reaction function smaller the size of the entrant relative to it. Thus a small entrant faces less competition from a large incumbent where large is said to be equivalent to well known. The 'largeness' is due to advertising expenditure which produces goodwill which should hence be curtailed to make entry by the small entrant less attractive. This indirect effect of advertising dominates the primary effect it has on increasing the market share of the incumbent thus reducing the potential market share of the entrant. The crucial element in this argument seems to be the small size of the entrant which may not always be the case.

Fudenberg and Tirole (1984) have shown that if post-entry price competition can deter an entrant, the incumbent could choose to restrict advertising to signal to the entrant that in the post-entry period it will compete vigorously on price. In their model the incumbent is a monopolist who advertises to create a captive market for the product. When entry takes place price competition occurs. Price discrimination is not possible in this situation since the captive consumers obtained in the first period will pay the same price as the new consumers. The larger the captive market obtained earlier (due to greater advertising) the higher will be the incumbent's post-entry price. Fudenberg and Tirole identify the "fat-cat" effect in the situation where incumbent will be less inclined to price competition if it has obtained a large captive market through vigorous advertising earlier. If tough post-entry price competition prevents entry the incumbent might find it profit-maximizing to maintain a "lean and hungry" look by underinvesting in advertising.

Spence (1980) conducted a detailed examination of the circumstances under which advertising can cause costs per unit of revenue to decline which in turn can create an entry barrier. He analyzed the relative profitability of firms of varying size holding industry revenues constant and shows that output and advertising are used in certain combinations. These combinations are determined by the elasticities that characterize the response of price, advertising and production cost. Since economies of scale are determined by a combination of all these elasticities, diminishing returns to scale for advertising could still mean declining costs as revenues increase. Spence remarked that this is analogous to production theory where the sum of the response elasticities determined the extent of returns to scale.

The empirical research regarding advertising has been concentrated mainly on its effect on profitability. However, there have been studies which investigate some of the other aspects of advertising such as its relationship with other marketing variables and its response to new entry. The impact of advertising on price has been investigated empirically by assessing how it affects the price sensitivity of consumers. Wittink (1977) combined time series and cross section data and showed that there was a positive relation between the two for the brand he considered. The limitation of this study was that it was not based on a comparison among different brands and could be a result specific to one brand. Eskin and Baron (1977) found support in favor of this positive relation based on an experimental study. They remark that by raising the position of the product in the preference ordering of the consumer and promoting the belief that no close substitutes for the product exist, a negative relation between advertising and price sensitivity could be observed.

Krishnamurthi and Raj (1985) showed that increased advertising lowers price sensitivity using panel data for an established, frequently purchased brand from an ADTEL advertising field experiment. The existence of the inverse relation can prevent potentially costly price promotions and competitive retaliation. The allocation implication of this relation is that if higher advertising can sustain a higher price, allocation for promotional activity can be lessened without ill-effects. Mitra (1990) has shown that advertising and price sensitivity are negatively correlated in an experimental setting where different choice environments and different types of advertising are used. It is shown that in a stimulus based environment the relation between the two exists for both reminder (information about existence) and differentiating (stressing product differentiation) advertising. In a memory based choice environment the correlation turns out to be negative only for differentiating advertisements. These results are important since they indicate the relevance of the type of advertising with regard to its effect on price sensitivity.

Cowling and Cubbin (1971) investigated the relationship between price, advertising expenditure and demand after adjusting price for quality. Past sales were assumed to affect current advertising expenditure and the data relate to cars in the UK between 1957 and 1968. Prices were adjusted using cross-sectional variation in the prices and characteristics of cars. The quality adjusted prices were found to be significant determinants of market-share behavior, along with advertising. The study showed that firms set advertising levels based on sales revenue. In the short-run advertising is increased when market share drops.

The entry barrier effect of advertising is difficult to estimate because of the difficulties of getting advertising data at the exact time of entry. It is much easier to assess the response of firms in an industry after entry has occurred. Cubbin and Domberger (1988) used a two stage methodology to examine the advertising response to new entry by incumbent firms. The advertising responses for a sample of 42 companies in consumer goods markets was estimated using time series techniques. In the second stage cross section analysis is done to determine what caused the post-entry increase in advertising. The results indicate that more than a third of the incumbents made a significant positive advertising response to entry though more than 60% failed to respond at all. Cubbin and Domberger also found that advertising response was far more likely in a growing market than in a static one since firms are still trying to establish market position. The larger firms appeared to be more likely to respond than the smaller ones. Entry turned out to be successful in all cases to the extent that the entrant brands persisted after two years. This study suffers from the limitation of being concerned mainly with the diversification type of entry by existing firms rather than entry of new firms into the market. Established firms can often stay in a market while allowing a loss because of their diversified financial strength and cannot be easily forced to leave a market by excessive advertising or any other entry deterring device.

Early studies seeking a relation between advertising and sales revenue or profit always treated advertising as a current expense. Theoretically we know that goodwill can accumulate with advertising which stimulates sales in the future. Bloch (1971) used profit rates adjusted for the treatment of advertising as an investment and failed to find any significant relationship between the adjusted profit rates and advertising intensity. A simultaneous equation specification was suggested to correct for the fact that heavily advertised national brands could have a feedback from advertising to concentration through probable economies of scale. Strickland and Weiss (1976) constructed such a model where they had advertising intensity as a function of concentration, concentration as function of advertising intensity and a price-cost margin equation where the margin is a function of both concentration level and advertising intensity. They found that OLS and 2SLS estimates were consistent with each other. This implies that the simultaneous equations bias in the single equation models was not a serious one.

Gomes (1986) directly tested the competitive view of advertising versus the anticompetitive view. The primary hypothesis suggested by the anticompetitive theory of advertising is that with higher advertising, both large and small firms will enjoy higher profit rates. The reasoning behind this hypothesis is that advertising promotes product differentiation which creates a barrier to entry by reinforcing concentration. The competitive theory suggests that as advertising increases in an industry, the larger firms get higher profits and market shares while the smaller firms do not gain in either profit rates or market shares.

This hypothesis is based on the assumption that the larger firms advertise in order to capitalize on their efficiency of production. The empirical analysis of Gomes supports the competitive hypothesis that advertising serves the function of providing price, product and seller information which helps large firms realize their efficiency by increasing sales.

Eckard (1987) used a different hypothesis to test the significance of the competitive theory of advertising. The hypothesis he tested was that if advertising generates market power by reducing firm demand elasticity, cross elasticity among firms should also fall implying greater market share stability. Eckard found no evidence that the market share instability of leading firms is less in high-advertising industries relative to low-advertising industries. This result is thus consistent with the competitive view of advertising which implies that rivalry among firms in advertising-intensive industries is at least as strong as in other industries. Schroeter et al. (1987) investigated the competitive hypothesis in a service market. Data on attorney fees and advertising practices in seventeen metropolitan areas across the US was used to estimate the effect of market advertising intensity on firm demand elasticities holding other factors constant. This investigation of the professional service market supported the competitive theory of advertising by showing that advertising intensity can potentially bring about a considerable reduction in price-cost ratios.

A recent trend in the empirical literature is the investigation of noncooperative competitive behavior in oligopolistic markets. Specifically, various equilibrium outcomes of games in advertising and price between firms have been tested for. Gazmi and Vuong (1988) considered different game theoretic models where firms set prices and advertising levels either simultaneously or sequentially. The econometric model associated with each game was a simultaneous equations model with model specific nonlinear constraints on the parameters. The leadership (Stackelberg behavior) models were found to be significant in the soft drinks market where Coke and Pepsi were the firms under consideration. The econometric models that were used suffer from misspecification thus implying approximate first order conditions and thus possibly misleading conclusions.

Roberts and Samuelson (1988) developed a dynamic model of advertising competition for the cigarette industry using intertemporal conjectural variation. Advertising is modeled in its goodwill formulation where current advertising is assumed to contribute to future sales. Firms which recognize and account for the optimal future rival actions on current advertising are classified as sophisticated, while firms that do not are called naive. The empirical evidence suggests the use of sophisticated behavior by firms in both the low-tar and high-tar cigarette markets. The hypothesis of joint-profit maximizing choice of goodwill levels in both market segments was tested for and was not rejected.

Overall, the literature indicates that it is of great interest to firms to learn what should be the optimal level of advertising expenditure given that competitors exist in the market place. The modeling of firm behavior is complicated by the fact that the environment in which advertising strategy is to be decided in, is both dynamic and uncertain. In the following chapters we will analyze monopoly and duopoly models of advertising in such an environment. The theoretical models show optimal paths of advertising and price over time for a new consumer durable. The empirical model employs the conjectural variations approach to analyze advertising and pricing competition in an established duopolistic consumer product market. A lagged price and advertising reaction function is also estimated to test for dynamic noncooperative behavior which is a consistent way of viewing competitive reactions by firms. The results obtained in the next three chapters provide interesting insights into optimal dynamic advertising policies of firms.

# CHAPTER 3
# OPTIMAL ADVERTISING POLICIES

## Introduction

The formulation of a long-term advertising strategy for a new product is something that brand managers have to deal with quite often. They have to keep in mind that sooner or later there are going to be competitors in the market and their behavior will have to be modified to account for this competitive interaction. The decision making is usually done under uncertainty since the time of entry of rivals in the market cannot be accurately predicted. Even if the firm knows that it will be facing competition at a given time there is a question as to what it should do to optimize its profits given such information. Since advertising interacts with other marketing mix variable like price, a firm needs to know the nature of this interaction in order to set its advertising and its pricing policy optimally. The consequences of manipulating price and/or advertising with the hope of discouraging entry is another important issue.

The objective of this research is to analyze the above much discussed issues in a continuous time framework in both monopoly and duopoly settings. While the static analysis provides a great deal of insights, it does not capture the strategic behavior of the firm over time. Typically a brand manager decides what strategies to follow over a planning horizon. Unless the manager sticks to some constant level of his choice variable, he will have to decide how to vary the variables over the planning horizon. This can only be captured in dynamic models. In fact this is the situation brand managers face everyday - a dynamic, uncertain and competitive environment in which to optimize their profits.

One of the earliest game-theoretic models was proposed by Shubik in 1961 and published later in Shubik (1980). The dynamics of advertising was modeled in the classic paper by Nerlove and Arrow (1962) where they discuss the goodwill creating aspect of advertising. Friedman (1983) used a discrete time analog of this model to analyse noncooperative behavior and finds conditions under which cooperative and predatory advertising could occur. One of the first differential game model of advertising was constructed by Case (1979) and was modified very recently by Sorger (1989). Both these papers successfully used Bellman's technique since only one state variable was modeled.

A number of papers have appeared in the industrial organization and marketing science literature which deal with the dynamics of duopolistic and oligopolistic models. Eliashberg and Jeuland (1986) study differences in the pricing decisions of foresighted, myopic and surprised monopolists facing certain entry in a subsequent period. Clarke and Dolan (1983) used simulation techniques to study a situation similar to that of Eliashberg and Jeuland. These papers are based on the monopolistic dynamic pricing literature which include Robinson and Lakhani (1975), Dolan and Jeuland (1981) and Kalish (1983) among others. Vidale and Wolfe (1957) discussed advertising response in the monopoly context which was later modified to reflect a competitive environment. Thompson and Teng (1984) discuss an interaction model of price and advertising in both monopolistic and oligopolistic contexts.

This chapter is most directly related to Thompson and Teng's work mentioned above. In Thompson and Teng's model dynamic demand is linear in advertising which leads to a bang-bang solution of the optimal control problem implying an optimal switching solution. In the model developed below, advertising enters the objective function in a nonlinear way. We initially assume that the gross margin is constant and the time of entry is certain. These assumptions are relaxed later to obtain more general insights.

The model presented in the second section deals with the advertising policy of a monopolist facing certain entry at a given time. In the third section the deterministic model in which both pricing and advertising are control variables is set up and analyzed. In the fourth section we briefly look at the case where the monopolist's dynamic demand is generalized to be a non-linear function of price.

## Dynamic Models of Monopoly and Duopoly

A monopolist introducing a new product has to determine the nature of its short run pricing and advertising policies. Formulation of optimal long run policies is difficult because the nature of future competition is unknown. The monopolist knows that its monopoly reign will only last for a finite period of time within which the market must be exploited optimally. The dynamic policies show the direction in which variables such as advertising move as the untapped market diminishes in size. Of particular interest is the change in the optimal policy after a second firm enters the market.

The following are the dynamic demand equations or sales response functions under a monopoly and duopoly in this model are postulated respectively as follows:

$$\dot{x}_1 = (M - x_1)a_1 \tag{3.1}$$

$$\dot{x}_i = [m - (x - x_1(T_1))][a_i + \gamma(a_i - a_j)] \tag{3.2}$$

$i,j = 1,2$ , $i \neq j$ and $x = x_1 + x_2$

The notation for the above follows that of Eliashberg and Jeuland and will be used throughout the chapter consistently and is reproduced below:

$x_i$ cumulative sales of firm i; first entrant is firm 1 which enters at time zero, second entrant is firm 2.

$T_1$ time of second entry

$T_2$ time horizon of duopoly period

$dx_{i/dt}$ sales rate at time t, ($x_{i\ dot} = dx_i/dt$)

M total population size

$m = M - x_1(T_1)$

uncaptured market at the beginning of the duopoly period

$a_i$ advertising effectiveness of firm i

$q_i$ constant price cost margin of product i

$\gamma$ differential advertising effectiveness between two products

x total cumulative sales of the two firms, $x_1 + x_2$

Both $x_i$ and $a_i$ vary with time, but the time argument has been omitted for notational simplicity.

Before working with the model it is appropriate to discuss some of the special assumptions used. The dynamic demand equation used in the monopoly case is the simplest possible example of how advertising acts on the available market where there is only one purchase per customer with no repeat buying. Horsky and Simon (1983) have used a non-linear function of advertising to model diffusion. The simpler linear function is used here in the interests of tractability of the model. In the duopolistic case the difference term in the advertising of the two firms is introduced as an adaptation of the Vidale-Wolfe sales response function to the competitive case where advertising by a rival firm lead to a depreciation of the effect of a firm's advertising on its current sales. Little (1979) showed that the Vidale-Wolfe model can be thought of as a special case of the more general class of models called the Lanchester warfare models.

A constant price cost margin is assumed in order to focus on the advertising policy. Both the Case advertising game model and Sorger's modification of it, use this assumption for the same purpose. The certain time of entry of a new firm on the market is something which has been observed in industries where specialized factories have to be set up in order to manufacture the product. Due to the nature of the specialized capital investment, the incumbent firm can get a ballpark date of the time of entry. Both of these assumptions will be relaxed in the later chapters. The other information assumption made is that firms know each other's price cost margin and demand. The behavioral assumption made is that of noncooperative behavior during the duopoly period. Under these assumptions, the appropriate equilibrium concept is Nash. The open loop Nash equilibrium strategies are functions of time alone and do not depend on the current state of the game. Playing open loop or using open loop controls implies that the firms determine their actions in advance for the entire planning period i.e., they can make long-term commitments about their strategy variables. The Nash equilibrium concept is defined as the combination of strategy choices by a firms where each firm maximizes its own payoff with respect to his own strategy choice, given those of the other firms.

The objective function to be maximized by the monopolist is the following:

$$\text{Max} \int_0^{T_1} (q_1 \dot{x}_1 - \frac{1}{2} a_1^2)\,dt$$

This brings us to the important assumption that has been made regarding the cost of advertising. The cost is assumed to be non-linear, specifically it is a quadratic function of the advertising effectiveness. This implies that the cost of advertising effectiveness rises at an increasing rate with advertising effectiveness. The rationale for this is that once the early adopters and imitators have purchased the product effective advertising becomes less effective. Without a significant word of mouth effect, reaching the remaining market requires more advertising. So the cost of achieving the same increase in effective advertising becomes increasingly more expensive. The entire advertising campaign may need to be intensified or even changed to appeal to the stubborn segment. Direct empirical validation of the convexity of advertising costs is difficult because advertising effectiveness cannot be measured directly. Even if cost per message rises, the cost of advertising effectiveness could still be convex. This assumption is made in all the models. Following Eliashberg and Jeuland (1986), the profits are not discounted over time in the interests of mathematical tractability. The effect of profits being discounted is discussed in the context of uncertain entry in the next chapter. Since the objective function will be maximized over a finite horizon, convergence is not a problem.

The necessary conditions for optimality of the above objective function are obtained by maximizing the Hamiltonian:

$$H_1=q_1\dot{x}_1-\frac{1}{2}a_1^2+\dot{\lambda}_1\dot{x}_1=>$$
$$H_1=(q_1+\lambda_1)(M-x_1)a_1-\frac{1}{2}a_1^2$$

where $\lambda_1$ is the adjoint variable for the dynamic equation (3.1) and is interpreted as the marginal valuations of the state variable $x_1$ in terms of the objective function. This type of dynamic optimization technique has been extensively used in macroeconomics as well as in marketing science and is discussed in Kamien and Schwartz (1981) and Sethi and Thompson (1981). The first order conditions are:

$$\frac{\partial H_1}{\partial a_1}=0=>a_1=(q_1+\lambda_1)(M-x_1)$$
$$-\frac{\partial H_1}{\partial x_1}=\dot{\lambda}=>\dot{\lambda}_1=(q_1+\lambda_1)a_1$$

Using the above conditions the following proposition can be derived:

Proposition 3.1: The monopolist finds it optimal not to change advertising intensity from the level chosen at the beginning of the time period 0 to $T_1$, i.e. $da_1/dt = 0$.

Proof: Differentiating the advertising first order condition with respect to t we get:

$$\dot{a}_1=(M-x_1)\dot{\lambda}-\dot{x}_1(q_1+\lambda)$$

The result is obtained by substituting the adjoint condition in the above expression.

The advertising intensity is constant due to two reasons. First, the constant price cost margin without discounting of future profits means that the monopolist will earn the same per unit profit regardless of whether he sells a unit of the product today or later. With discounting of future profits, revenue earned at the present time is worth more. It would then be rational for the firm to advertising heavily in the beginning and decrease advertising over time. This intution is borne out in the next chapter when uncertain entry causes the monopolist to discount future profits. Secondly,the absence of competition means that the monopolist will make the sale to whoever wants to buy the product; if not today then tomorrow. This implies that there is no uncertainty about making the sale. If the monopolist attempts to force the pace by raising advertising above the optimal level, the quadratic cost (which increases faster than the advertising effectiveness) is going to eat into his profits. This result can be used to derive the optimal constant level of advertising of the firm. Integrating out the objective function, differentiating with respect to $a_1$ and setting the derivative equal to zero we get:

$$a_1 = \frac{q_1(MT_1 - \int_0^{T_1} x_1 dt)}{T_1} \qquad (3.3)$$

Integrating both sides of the dynamic demand equation and solving the resulting differential equation we get (detailed derivation given in the appendix):

$$x_1 = M(1 - \frac{1}{e^{a_1 t}}) \qquad (3.4)$$

Integrating both sides of (3.4) between 0 and $T_1$ we get (shown in the appendix):

$$\int_0^{T_1} x_1 dt = MT_1 - \frac{x_1(T_1)}{a_1}$$

Substituting the above in (3.3) we get:

$$a_1 = \sqrt{\frac{q_1 x_1(T_1)}{T_1}} \qquad (3.5)$$

An examination of (3.5) tells us the following:

a. Advertising does not depend directly on the population though it does so indirectly since $x_1(T_1)$ depends on M.

b. It varies directly with the square root of the price cost margin $q_1$.

c. The effect of the length of the time period is not clear because it has both a direct and an indirect effect through $x_1(T_1)$ which go in opposite directions. From (3.4) we can see that $x_1(T_{1)}$ increases with $T_1$. This means that with a longer time period the firm will penetrate the market to a greater degree. In (3.5), which is an implicit expression defining $a_1$, $T_1$ enters the denominator directly thereby reducing $a_1$ as it increases. These two effects taken together produce an ambiguous effect of $T_1$ on advertising. The average sales per unit time drives the optimal level of advertising here. The higher the level of average sales over the time horizon, the greater will be the optimal level of advertising for the monopolist.

Substituting (3.5) in the integrated out objective function we get the following expression for optimal profits:

$$\Pi_1^{*}=q_1 x_1(T_1)\left(M-\frac{1}{2}\right) \tag{3.6}$$

From (3.6) we can see that the optimal profits are a linear function of market penetration $x_1(T_1)$, since both $q_1$ and (M - 1/2) are constants.

We now consider what happens when a second firm enters the market at time $T_1$ and a duopoly prevails till time $T_2$. The relevant dynamic demand equation will now be (3.2) where the rate of change of sales will depend on the difference in the advertising effectiveness levels of the two firms as well as the own advertising effectiveness of the firm. Also the unpenetrated or available market term now reflects that there are two firms obtaining sales now. The optimization problem for the firms will now be formulated as a differential game which can be solved in principle to obtain the open loop Nash equilibrium strategies. The objective function to be maximized and the Hamiltonian for each firm are the following:

$$\int_{T_1}^{T_2}(q_i\dot{x}_i-\frac{1}{2}a_i^2)\,dt$$

$$H_i=q_i\dot{x}_i-\frac{1}{2}a_i^2+\lambda_{ii}\dot{x}_i+\lambda_{ij}\dot{x}_j$$

$$H_i=(q_i+\lambda_{ii})(m-x)[a_i+\gamma(a_i-a_j)]+\lambda_{ij}(m-x)[a_j+\gamma(a_j-a_i)]-\frac{1}{2}a_i^2$$

where i,j = 1,2 and $i\neq j$.

The first order conditions are obtained as in the monopoly case. Following Eliashberg and Jeuland (1986) zero salvage value is assumed at the end of the planning horizon $T_1$ to $T_2$ which results in the boundary conditions $\lambda_{ii}(T_2)=\lambda_{ij}(T_2)=0$. It is easy to verify that $\lambda_{ij}(T_2) = 0$ and $d\lambda_{ij}/dt = -(\partial H_i/\partial x_j)$ implies $\lambda_{ii} = \lambda_{ij}$ and we will define $\lambda_i = \lambda_{ii} = \lambda_{ij}$ where $i\neq j$ and i,j = 1,2. Using the first order conditions we can derive the following result:

Proposition 3.2: The optimal advertising strategy for each duopolistic firm declines over time and the rate of decline is given by

$$\dot{a}_i=-[m-x]q_i(2\gamma+1)a_j \qquad (3.7)$$

Proof: See the appendix.

The intuition behind proposition 3.2 is that the advertising effectiveness decreases over time due to the presence of the second firm. The size of the remaining market decreases faster since today's untapped market may not be available tomorrow. A consumer not captured now may be captured by the rival in the future. Equation (3.7) demonstrates that the rate of decrease of advertising effectiveness varies directly with:

a. the potential revenue from the unpenetrated portion of the market i.e. $q_i[m - x]$

b. the advertising effectiveness level of the rival firm since this will increase $x_j$ and thereby decrease the untapped market, $[m - x]$.

If we substitute $a_j = 0$ in (3.7) above we get $da_i/dt = 0$ which is nothing but proposition 3.1 derived in the monopoly case. The optimal level of advertising wil also be the same if $\gamma$ is set to zero in keeping with the monopoly market environment. Proposition 3.2 also yields the following result:

$$\frac{\dot{a}_i}{\dot{a}_j} = \frac{\frac{a_i}{a_j}}{\frac{q_i}{q_j}}$$

which says that the ratio of the rates of change of the advertising effectiveness is equal to the ratio of relative advertising intensities and the price cost margins.

Horsky and Simon (1983) obtained a result similar to proposition 3.2 in the monopoly case which validated Mate's (1980) numerical simulation result. A related result appears in Erickson (1985) where power functions were used to model competitive advertising in a differential game framework for an established product. Erickson found that a low but increasing market share for a firm will result in a situation in where the firm's advertising is relatively high and declining while the rival firm's advertising is low and increasing.

Eliashberg and Jeuland (1986) define a myopic monopolist as one who views the information that there will be entry at time $T_1$ as a signal to get the maximum profits before entry not recognizing that opportunities for profit exist in the subsequent duopoly period. A non-myopic or foresighted monopolist is defined as one which seeks to maximize profits over the current monopoly period and the upcoming duopoly period. A myopic monopolist is clearly differentiated from a naive one who does not react to the information about the certain entry at all. The Elaishberg and Jeuland model gives the result that in a two period horizon with duopoly following monopoly, the non-myopic monopolist prices higher than a myopic one and decreases his price less rapidly. In the case of advertising with a constant price cost margin the question is whether or not the foresighted monopolist will find it optimal to follow the same constant level of advertising implied by (3.5) as the myopic monopolist.

The foresighted monopolist has to take into account that the future market will be a shrinking and a shared one. The optimal advertising policy of the foresighted monopolist can in principle be derived by maximizing joint profits over the monopoly and duopoly horizons. The time of entry $T_1$ and the level of the rival firm's advertising determines the optimal policy the foresighted monopolist will follow. With an early impending entry of a second firm, it will be optimal to advertise heavily to achieve maximum market penetration. The forsighted monopolist will recognize that opportunities for profit exist in the duopoly period and accordingly will not follow the same advertising policy as the profit maximizing myopic monopolist. If the rival firm advertises at a high rate (depending on its price cost margin) in the duopoly period the optimal rate of advertising will decrease faster according to (3.7). This could lead to lower advertising costs in the duopoly period thereby increasing duopoly period profits. The exact nature of the advertising policy of the foresighted monopolist cannot be derived without solving the complicated two period joint profit maximization problem.

## Price Advertising Interaction Model

In this section we relax the assumption that advertising is the only choice variable for the monopolist and duopolist. The dynamic demand equation used here is:

$$\dot{x}_i=(M-x_1)a_1(1-kp_1) \tag{3.8}$$

where

| | |
|---|---|
| $p_1$ | price |
| k | price sensitivity, and |
| $c_1$ | constant marginal cost. |

Equation (3.8) is identical to the dynamic demand equation used by Eliashberg and Jeuland except for the substitution of the diffusion parameter by advertising effectiveness. The TRACKER model of Blattberg & Golanty (1978) closely resembles (3.9) which represents the product of a growth factor and the static demand function of the form a - bp.

Considering the monopoly case first we perform dynamic optimization using the Hamiltonian approach. The objective function is:

$$\text{Max} \int_0^{T_1}[(p_1-c_1)(M-x_1)(1-kp_1)a_1-\frac{1}{2}a_1^2]dt$$

The Hamiltonian is written as before and the first order conditions obtained by differentiating the Hamiltonian with respect to price in addition to advertising. The multiplier equation is obtained as before.

$$H_1=(p_1-c_1+\lambda_1)(M-x_1)(1-kp_1)a_1-\frac{1}{2}a_1^2$$

$$\frac{\partial H_1}{\partial p_1}=0=>p_1=\frac{1}{2}(\frac{1}{k}+c_1-\lambda_1)$$

$$\frac{\partial H_1}{\partial a_1}=0=>a_1=(p_1-c_1+\lambda_1)(1-kp_1)(M-x_1)$$

$$\dot{\lambda}_1=-\frac{\partial H_1}{\partial x_1}=0=>\dot{\lambda}_1=(p_1-c_1+\lambda_1)(1-kp_1)a_1$$

Using the above first order conditions we obtain the following expressions:

$$a_1=(M-x_1)\left(\frac{1}{k}-2p_1+kp_1^2\right) \tag{3.9}$$

$$\dot{p}_1=-\frac{1}{2}\left(\frac{1}{k}-p_1\right)(1-kp_1)a_1 \tag{3.10}$$

Proposition 3.3: The monopolist who can vary price and advertising, sets advertising optimally at a constant level as in the case where advertising was the only strategic variable.

Proof: The result follows from the expression for $da_1/dt$ obtained by differentiating both sides of (3.9) with respect to time and substituting (3.10) in it. The detailed derivation is shown in the appendix.

The intuition behind this proposition remains the same as what was discussed for proposition 3.1 because price enters the dynamic demand equation linearly. In the absence of a non-linear interaction term between advertising and price, the nature of the advertising control does not differ from the case where it was the sole control variable. The interesting feature of this result is that optimal advertising is constant even though the objective function is non-linear in price.

The immediate consequence of proposition 3.3 is that a problem with two control variables reduces to that of only one. Given the fact that advertising will not vary over time we can derive a solution for the optimal value of price given the level of penetration at the end of the period. In effect Eliashberg and Jeuland's monopolist problem is the same as ours even though both advertising and price are control variables for the firm. The following is the expression for the monopolist's price which can be obtained utilizing the first order conditions stated above given the fact the advertising level is to be treated as a constant now:

$$p_1^* = \frac{1}{k}\left[1 - \frac{2}{a_1\left[\dfrac{T_1}{1-\sqrt{1-x_1/M}} - t\right]}\right]$$

This equation has been derived in the Appendix B of Eliashberg and Jeuland (1986) and is the result stated in proposition 2 of their paper. We can see from the expression above that a higher constant level of advertising will imply a higher optimal price at any given time t. Since higher advertising implies greater market penetration for the firm, the marginal valuation of the untapped market will be lower. Given the first order condition for price, the lower marginal valuation $\lambda_1$ will lead to a lower optimal price.

Proposition 3.4: At higher price levels, the price decreases faster over time than at lower prices for the monopolist.

Proof: Follows directly from (3.10).

Proposition 3.4 is reasonable since at higher prices falling demand would imply a greater loss of revenue for the firm which would then cut prices quickly to counter the decreased demand. The relationship between advertising and rate of change of prices is also indicated by (3.10) which tells us that prices fall faster over time with high advertising.

Differentiating (3.9) w.r.t. k we obtain:

$$\frac{\partial a_1}{\partial k} = (M-x_1)(p_1^2 - \frac{1}{k^2}) \qquad (3.11)$$

Proposition 3.5: The optimal advertising is inversely related to price sensitivity for the monopolist.
Proof: Equation (3.11) will be positive or negative depending on the sign of ($p_1$ - 1/k) which has to be negative to ensure that demand is non-negative.

This result has empirical support as discussed in chapter two though not all studies have unequivocally supported the assertion. The magnitude of the effect of advertising on price sensitivity is small and hence is difficult to quantify empirically. It is possible that experimental studies may be the direction to go in order to find under what conditions a positive relation or a negative one is obtained.

Equations (3.9) and (3.11) both have ($M-x_1$) entering multiplicatively on the right hand side. This will of course be positive as long as the market is not exhausted which means it will be positive as long as the firms can operate in the market. The higher the market penetration $x_1$, the smaller is the size of the unpenetrated market and the lower is the rate of decline of the price over time, the greater the magnitude of the inverse variation between advertising and price and between advertising and price sensitivity.

We will now consider a dynamic duopolistic model where each firm controls both price and advertising. Thompson and Teng (1984) considered such as a problem in a N firm scenario where there is a single price set by the largest competitor -- a price leadership framework. They consider two cases, (a) linear prices and (b) exponential function of prices. These assumptions did not allow them to derive analytical results and forced immediate recourse to simulation. Keeping in mind the tractability problems we make the following assumptions in order to derive some analytical results:

a. N=2 i.e. duopoly
b. no salvage value after terminal time, and
c. no discounting over time.

The dynamic demand equation that will be employed is:

$$\dot{x}_i=(m-x)[a_i(1-kp_i)+\gamma(a_i-a_j)] \qquad (3.12)$$

The duopoly dynamic equation is the same as in the last section except for the presence of the (1 - $kp_i$) term which involves price. We can see that the slope of the demand curve depends on own advertising of the firm but the position depends on the difference between the advertising levels of the two firms. The Hamiltonian and the first order conditions are obtained as before and are shown below:

$$H_i=(p_i-c_i+\lambda_i)(m-x)[a_i(1-kp_i)+\gamma(a_i-a_j)]-\frac{1}{2}a_i^2+\lambda_i(m-x)[a_j(1-kp_j)+\gamma(a_j-a_i)]$$

$$\frac{\partial H_i}{\partial p_i}=0=>p_i=\frac{1}{2}[\frac{1}{k}+c_i-\lambda_i]$$

$$\frac{\partial H_i}{\partial a_i}=0=>a_i=(p_i-c_i)(m-x)(1-kp_i+\gamma)+\lambda_i(m-x)(1-kp_i)$$

$$\dot{\lambda}_i=-\frac{\partial H_i}{\partial x_i}=(p_i-c_i+\lambda_i)[a_i(1-kp_i)+\gamma(a_i-a_j)]+\lambda_i[a_j(1-kp_j)+\gamma(a_j-a_i)]$$

where $i \neq j$.

The first order condition can be manipulated to yield the following relation:

$$a_i=(m-x)[\frac{1}{k}+(\gamma-2)p_i+kp_i^2-\gamma c_i] \qquad (3.13)$$

Differentiating (3.13) w.r.t. k we get:

$$\frac{\partial a_i}{\partial k}=(m-x)(p_i^2-\frac{1}{k^2}) \qquad (3.14)$$

We can immediately see the similarity of (3.11) obtained in the monopoly case with (3.14) obtained above. The unpenetrated market term is now redefined to include the second firm's cumulative sales. On the basis of (3.14) proposition 3.5 can be immediately extended to the duopoly case.

Proposition 3.6: Proposition 3.5 is also true for the duopoly case.

Proof: The same argument as for proposition 3.5 applied to (3.14) yields the result.

Differentiating the first two first order conditions with respect to time and collecting terms gives the expressions for $dp_i/dt$ and $da_i/dt$ below:

$$\dot{p}_i=-\frac{1}{2}(P_i-c_i)[a_i(1-kp_i)+\gamma(a_i-a_j)]-\frac{1}{2}(1/k+c_i-2p_i)[a_i(1-kp_i)+a_j(1-kp_j)]$$

$$\dot{a}_i=-(m-x)[a_j(p_i-c_i)((1-kp_i+\gamma)(1-kp_j)+\gamma(1-kp_i))-\gamma\dot{p}]$$

On the basis of the above expressions we can state the following proposition:

Proposition 3.7: In a duopoly setting, it is optimal for firms to decrease advertising over time as long as price is falling over time.

Proof: This result follows from the examination of the expressions given above.

The proposition implies that when prices are being reduced over time, advertising is being reduced over time as well in order to optimize profits in a situation where the remaining market is shrinking over time. The maintenance of a high rate of advertising would reduce profits because of the quadratic costs of advertising.

The adjoint variable is zero at the end of the horizon by assumption. Under symmetry assumptions, it can be shown that the adjoint variable increases over time (follows from the adjoint condition above). This implies that the adjoint variable or the marginal valuation of cumulative sales is negative through the time horizon. Since the untapped market shrinks over time the marginal valuation decreases over time. Though we cannot demonstrate this result in the general case we may expect the marginal valuation to remain negative due to the continuous saturation of the market over time.

## Dynamic Demand as a Nonlinear Function of Price

The dynamic demand equation that has been used so far is a linear function of price. An alternative to this function is using the power function (i.e. the Cobb-Douglas type of function) where the exponent of advertising is 1 which implies unit advertising elasticity. In the dynamic equation used here $\eta$, the exponent of the price term, is interpreted as the price elasticity of demand. This function has appeared in a number of models including the TRACKER model of Blattberg & Golanty (1978). We will see if propositions similar to propositions 3.5 and 3.6 can be derived for this type of dynamic demand equation. This would serve to show how sensitive the propositions are to the form of the dynamic demand equation.

The following is the power function that will be used in this section:

$$\dot{x}_1 = (M-x_1)\, a_1 p_1^{\eta} \tag{3.16}$$

The Hamiltonian and the first order conditions are obtained as before and is shown below:

$$\begin{aligned}
H_1 &= (p_1-c_1+\lambda_1)(M-x_1)\, a_1 p_1^{\eta} - \frac{1}{2} a_1^2 \\
\frac{\partial H_1}{\partial a_1} &= 0 \Rightarrow a_1 = (p_1-c_1+\lambda_1)(M-x_1)\, p^{\eta} \\
\frac{\partial H_1}{\partial p_1} &= 0 \Rightarrow p_1 = \frac{\eta}{1+\eta}(c_1-\lambda_1) \\
\dot{\lambda}_1 &= -\frac{\partial H_1}{\partial x_1} \Rightarrow \dot{\lambda}_1 = a p_1^{\eta}(p_1-c_1+\lambda_1)
\end{aligned}$$

Proposition 3.8: Proposition 3.3 holds in the case when dynamic demand is a non-linear function of price.
Proof: See the appendix.

Proposition 3.8 shows that a nonlinear Cobb-Douglas type of dynamic demand does not alter the nature of the monopolist's optimal advertising policy which is maintenance of a constant level of advertising. It is interesting to note that $\eta$, which is as the price elasticity of dynamic demand, does not affect the optimal advertising policy. The effect of $\eta$ on the optimal level of advertising is of interest and is the subject of the next proposition.

Proposition 3.9: The monopolist's optimal advertising varies inversely with the price elasticity of demand.
Proof: The result follows directly from the expression for $\partial a_1/\partial \eta$, keeping in mind that $a_1$ and $p_1$ are positive while $\eta$ is negative. The details of the proof are shown in the appendix.

In the model presented in the second section of this chapter, the comparable proposition is 3.5. The price sensitivity parameter k, in the linear case is roughly equivalent to the price elasticity parameter $\eta$ here. Intuitively this proposition states that when consumer demand becomes more price elastic, the monopolist will reduce the use of advertising in his marketing mix. This appears to be a logical course of action again keeping in mind the convex advertising costs. The results of this section indicate that the results obtained in the second section are not sensitive to the linear function of price specification of the dynamic demand function.

# CHAPTER 4
# STOCHASTIC MODELS OF ADVERTISING

## Introduction

In the models of the previous chapter firms determined optimal policies on the basis of a certain planning horizon. One of the objectives of this chapter is to introduce an uncertain time of entry and analyze its effects on the decisions of the monopolist and duopolists. In general a potential entrant is unlikely to reveal its intentions sufficiently far in advance to the incumbent such that it can account for the entry in its decision making. Quite often the entrant is an established firm in a related product market which makes it easy for it to disguise its intentions from the incumbent firm.

Kamien and Schwartz (1971) have dealt with this problem of uncertain entry. In formulating their model they draw on Williamson's (1963) stochastic generalization of the limit price concept. Following Stigler (1968) they postulate that industry growth encourages entry. They assume that the conditional probability of entry at time t given that entry has not occurred yet, is an increasing function of product price at time t as well as the industry growth rate. Kamien and Schwartz (1971) show that the firm's optimal policy (maximizing present value of total expected profits), is to set the price at some appropriate constant level.

Bourguignon and Sethi (1981) have extended the Kamien and Schwartz model to the case the where the conditional probability of entry depends on the probability that entry has occurred before time t as well as the prevailing product price and industry growth rate. They point out that it is quite possible that there will be some potential entrants who are better informed and have more information than just the current price set by the incumbent firm. They find that Kamien and Schwartz (1971) results can be extended to more general stochastic processes of entry or to models where advertising can be used as an entry deterrent. One of the limitations of their analysis is that the entrant considers current monopoly profits instead of post-entry profits when making its entry decision.

Another relevant source of uncertainty is the presence of a stochastic element in the dynamic demand equation itself. This is precisely the situation that real world managers have to confront, because they are never certain about the magnitude of the effect of their marketing variables (price and advertising in our case). The dynamics of demand is commonly modeled by specifying a differential equation for the growth of cumulative sales as has been done in the previous chapter. In order to include the effect of uncertainty on the growth rate, we introduce a random error term into the state equation, thus converting it into a stochastic differential equation. The cumulative demand then evolves as a stochastic process which can be influenced probabilistically by the control variables such as advertising. The monopolist's problem now becomes a stochastic optimal control problem and the duopoly problem amounts to a stochastic differential game.

In the second section of this chapter we discuss a model where a myopic monopolist faces an uncertain time of entry. In the third section we analyze the situation where a firm engages in entry postponing behavior while the last section deals with the stochastic optimal control problem of the monopolist and the stochastic differential game model of a duopoly.

## Effect of Uncertain Time of Entry

We assume that there is a constant hazard rate for the event of entry. Thus, if $T_1$ is the random entry time then, prob (entry at "t", given no entry up to t) = $F'(t)/(1-F(t)) = r$, where F denotes the cumulative distribution function of $T_1$. Assuming that $T_1$ is continuous, it follows that F(.) is differentiable, and we can obtain the probability density function. More generally, if $F'(t)/(1-F(t)) = h(t)$ then, it is well known that,

$$F(t)=1-\exp\left(-\int_0^t h(s)ds\right).$$

Hence our assumption of constant $h(t)\equiv r$ implies an exponential entry time:

$$F(t)=1-\exp(-\lambda t)$$
$$f(t)=F'(t)=\lambda\exp(-\lambda t)$$

Kamien and Schwartz (1971) and Bourguignon and Sethi (1981) used the hazard rate concept to model stochastic entry. Hazard rates have played an important role in marketing science as exemplified by the well-known example of their application to new product growth models in Bass (1969). The probability of entry changes once one or more entries have occurred. The hazard rate is the appropriate modeling tool to use since we are interested in the conditional probability of the occurrence of the first entry. The dynamic demand equation that will be used in this model is the same as the one used in the price advertising interaction model of the third section of the previous chapter:

$$\dot{x}_1=\frac{dx_1}{dt}=(M-x_1)a_1(1-kp_1)$$

where $p_1$ = price

$k$ = price sensitivity,

$c_1$ = constant marginal cost,

$M$ = constant population size,

$a_1$ = advertising effectiveness and

$x_1$ = cumulative sales.

The dynamic demand above is the same as the one used by Eliashberg and Jeuland (1986) except that the diffusion parameter depends on advertising effectiveness. The equation represents the product of a decaying growth factor and the static demand function. The assumption of convex cost of advertising made in the deterministic models of the previous chapter is retained here. Gould (1970) and Case (1979) among others have employed a similar advertising cost structure. Recalling that we identified two kinds of monopoly behavior, we will restrict our attention to the case where the monopolist is myopic by as defined in Eliashberg and Jeuland (1986). By definition, the myopic monopolist is not concerned with the maximization of post-entry duopoly profits. Instead, he focuses only on maximizing his profits prior to the event of entry and does not recognize that opportunities for greater total profits across periods can exist.

Under the constant hazard rate assumption, the time of entry $T_1$ is now an exponentially distributed variable. Given that the length of the monopoly period T1 is a random variable, the objective function is now stochastic. The exponential distribution for $T_1$, allows us to rewrite the objective function by appealing to Merton's (1971) theorem VI as follows:

$$E\int_0^{T_1}[(p_1-c_1)\dot{x}_1-\frac{1}{2}a_1^2]dt=\int_0^{\infty}e^{-rt}[(p_1-c_1)\dot{x}_1-\frac{1}{2}a_1^2]dt$$

Merton's theorem allows us to transform a finite horizon problem into an infinite horizon deterministic problem.

The proof of Merton's theorem is provided in his original paper. In the appendix, we prove an extension of Merton's theorem which will be used in the next section. The verbal interpretation of the result is that when entry is uncertain, the incumbent firm places a lower value on profits obtained in the future since they are uncertain. Note that if we already had a discount factor in our problem, r would simply have been incremented by the amount of the discount factor. Hence, without loss of generality we may assume that the effect of temporal preferences is already reflected in r. Following Merton's (1971) interpretation, we can say that the incumbent firm which faces an exponentially distributed uncertain length of monopoly acts as if entry will not occur, but has a subjective rate of time preference equal to the risk of entry. The risk of entry is the reciprocal of the expected value of the length of the monopoly period.

The formulation of the dynamic optimization problem is the same as before and the Hamiltonian for the problem can be written as:

$$H=e^{-rt}[(p_1-c_1)\dot{x}_1-\frac{1}{2}a_1^2]-\lambda\dot{x}_1$$

Following Kamien and Schwartz (1981, p.151) we rewrite the above and derive the corresponding first order conditions as follows:

$$
\begin{gathered}
H_c = e^{rt}H = (p_1 - c_1 + \lambda_c)(M - x_1)(1 - kp_1)a_1 - \frac{1}{2}a_1^2 \\
\frac{\partial H_c}{\partial p_1} = 0 \Rightarrow p_1 = \frac{1}{2}\left(\frac{1}{k} + c_1 - \lambda_c\right) \\
\frac{\partial H_c}{\partial a_1} = 0 \Rightarrow a_1 = (p_1 - c_1 + \lambda_c)(1 - kp_1)(M - x_1) \\
\dot{\lambda}_c = r\lambda_c - \frac{\partial H_c}{\partial x_1} \Rightarrow r\lambda_c + (p_1 - c_1 + \lambda_c)(1 - kp_1)a_1
\end{gathered}
$$

where $H_c$ is called the current value Hamiltonian and $\lambda_c = e^{rt}\lambda$ is the current value multiplier associated with it. $\lambda_c$ above is interpreted as the marginal value of the state variable at time t in terms of values at t.

Differentiating the first order conditions with respect to time and rearranging terms we get the following expressions for $dp_1/dt$ and $da_1/dt$:

$$
\begin{gathered}
\dot{p}_1 = -\frac{1}{2}\left[\left(\frac{1}{k} - p_1\right)(1 - kp_1)a_1 + r\left(\frac{1}{k} + c_1 - 2p_1\right)\right] \\
\dot{a}_1 = r(M - x_1)(1 - kp_1)\left(\frac{1}{k} + c_1 - 2p_1\right)
\end{gathered}
\tag{4.1}
$$

Proposition 4.1: For a myopic monopolist faced with an uncertain potential entry, it is optimal to decrease advertising over time so long as $(1/k + c_1 - 2p_1) < 0$ holds.
Proof: Follows from examination of (4.1) above.

What the above proposition implies is that as long as the current value marginal valuation of cumulative sales is negative, the monopolist will advertise at a decreasing rate in order to obtain more sales. Unlike the certainty case advertising no longer remains constant. The constant rate of advertising result is recovered when $r = 0$, which corresponds to the removal of uncertainty of entry occurrence. We can infer from this result that in the deterministic case of the previous chapter discounted profits would have meant decreasing advertising over time instead of keeping it fixed.

Horsky and Simon (1983) obtained the result that a monopolist with a new product decreases advertising as sales increase in the presence of diffusion effects. In our model, the possibility of entry in an uncertain environment drives the monopolist to adopt the same decreasing advertising over time policy in the case of a negative marginal valuation of cumulative sales even without the presence of any positive word of mouth effects. The marginal valuation of cumulative sales is expected to be negative since there is a fixed population which buys only a single unit of the durable good within the time horizon under consideration. The optimal pricing policy cannot be clearly characterized depending on the marginal valuation because it additionally depends on the advertising level. Examination of the first part of (4.1) above shows that pricing is likely to decrease over time at high levels and increase at low levels of advertising.

With decreasing advertising over time, price will initially decrease over time but will increase when the following condition (derived from the first equation in 4.1) involving the rate of discount, the price sensitivity parameter, price and cost is satisfied:

$$a_1 < \frac{r[1-k(2p_1-c_1)]}{(1-kp_1)^2}$$

The uncertainty of the length of its monopoly run necessitates the use of both marketing instruments simultaneously while the above condition is not satisfied. Price will increase over time if high discount rate coupled with a low price cost margin begins to drive profits to suboptimal levels. Thus it appears that when entry time is uncertain it is optimal for a new product myopic monopolist to maintain low prices with higher levels of advertising while increasing prices should accompany low advertising which prevail at very high levels of market penetration.

The infinite horizon and autonomous (i.e. t does not enter anywhere explicitly) nature of the problem makes it appropriate to examine the nature of the steady state which is defined as

$$\dot{x}_1 = \dot{\lambda}_c = 0$$

where

$$\dot{x}_1 = a_1(M-x_1)(1-kp_1)$$

It is the infinite horizon transformation of the original problem that makes it autonomous. With a finite horizon, time dependence persists through the constraint imposed by the boundary conditions at $T_1$. Using the above two expressions we get $x_1 = M$. This is expected since the cumulative sales will grow as long as all the population has not bought the product. The value of r does not affect the steady state outcome which is expected given that $x_1$ denotes cumulative sales. Note that this steady state will not be reached should entry by another firm actually occur. In the event of an entry the situation will revert to the duopoly differential game of the deterministic case. The comparable result obtained in the duopoly case is $m = x$ where the steady state is obtained by making the rate of change of cumulative sales with respect to time of both firms equal to zero as well as that of adjoint variable. Given the symmetric nature of the problem at steady state each firm will have a market share of m/2 and the entire market will be penetrated.

## Entry Postponing Behavior of Monopolist

Clarke and Dolan (1984) used simulation techniques to compare skimming versus penetration pricing policies in the same kind of two period framework as Eliashberg and Jeuland (1986). In his comments on their paper Friedman (1984) remarks that instead of considering a monopoly period followed by certain entry resulting in duopoly it may be better to deal with three separate models of monopoly, duopoly and monopoly with a potential entrant. Friedman feels that the scenario in which one firm enjoys a monopoly reign followed by certain entry at a point in time is artificial in nature.

We discussed the first two at length in the previous sections and the third situation is what we focus on now. As in the previous section, the time of entry is a random variable with an exponential distribution. The incumbent firm could attempt to lengthen its monopoly run if it thinks that it can affect r, the parameter of the exponential distribution of $T_1$, by advertising more or pricing lower, i.e. if r decreases with increase in advertising or decrease in price. The industrial organization literature discussed in the second chapter provides examples of models where advertising is used as an entry barrier. Waterson (1984) remarks that one way by which advertising might create barriers to entry is by creating durable preferences for the incumbents product which are hard and expensive to change. Cubbin (1981) shows that an entry barrier effect can exist as long as the entrant's prospective demand curve is made less favorable by an established firm's present advertising. We will develop below a model, where the firm advertises high under the belief that it can postpone entry on the part of the potential entrant.

We will use an extension of Merton's theorem used in the last section. The extension states that Merton's result also applies in the case where r, the parameter of the exponential distribution of the event of entry, is a function of time. The proof for this extension of Merton's result is provided in the appendix. We require this generalized result because r will now be a function of advertising which varies over time. The possible role of price as an entry deterrent has been discussed in Eliashberg and Jeuland (1986). We do not consider limit pricing here since experience effects or first-mover advantages are not captured in our dynamic demand equation. The objective function for this model is:

$$\mathrm{Max}\int_0^\infty e^{-\int_0^t r(a_1(s))ds}[(p_1-c_1)(M-x_1)(1-kp_1)a_1-\frac{1}{2}a_1^2]dt$$

The Hamiltonian and the first order conditions are stated below:

$$H=e^{-R}[(p-c)(M-x_1)(1-kp)a-\frac{1}{2}a^2]+\lambda(M-x)(1-kp)a$$

$$\frac{\partial H}{\partial a}=0=>(1-r(a)a)[(p-c)(M-x)(1-kp)]e^{-R}-ae^{-R}+\frac{1}{2}a^2r(a)e^{-R}+\lambda(M-x)(1-kp)=0$$

$$\frac{\partial H}{\partial p}=0=>e^{-R}[\frac{1}{k}-2p+c]=\lambda$$

$$\dot{\lambda}=-\frac{\partial H}{\partial x}=>\dot{\lambda}=(p-c)(1-kp)ae^{-R}+\lambda(1-kp)a$$

$$R=\int_0^t r(a(s))ds$$

where subscripts are neglected for notational convenience. The first order conditions above are used to derive the following expressions:

$$\begin{gathered}\dot{p}=-\frac{1}{2}k(1-kp)^2a \\ (M-x)(1-kp)[\frac{1}{k}-p-r(a)a(p-c)]=a-\frac{1}{2}a^2r(a)\end{gathered} \tag{4.2}$$

Proposition 4.2: It is optimal for the monopolist using advertising as an entry deterrent, to decrease price over time.

Proof: Follows from expression for dp/dt in (4.2).

Advertising now serves both to penetrate the market and postpone entry by a potential entrant. The incumbent is now likely to advertise more than he did in the model of the previous section. The price falls more quickly since now with the quicker penetration of the market due to the increased advertising, the firm is reaching the customers who are willing to pay less money for a new product. In other words, the price elasticity is now increasing over time and this results in price falling over time. Therefore advertising must decrease beyond a certain point. Note however that an increase in cumulative sales x reduces the effect of advertising regardless of whether the level of advertising is high or low.

Let us briefly consider the case of a duopoly faced by a potential entrant. Here r could be a function of the total amount of advertising $a_1 + a_2$ of the two incumbents. However this sort of function would not really fully capture the firms' behavior should one of them decide that it can free ride on the entry postponing behavior of the other. It would be rational for a firm to allow another to free ride on its entry deterring activity only as long its profits under triopoly are less than in a duopolistic market where it is the only firm advertising to deter entry.

Friedman (1983) remarked that if the entrant firm reasons rationally, it will realize that in the post-entry situation the incumbent will no longer have any reason to sustain the higher advertising and/or lower prices. If the incumbent firm continues to maintain the barriers after the occurrence of entry it will not be maximizing profits. The incumbent may elect to continue its entry deterrence policies in order to try and drive the entrant out of the market. Whether the entrant survives or not in the absence of substantial learning effects is likely to be a function of the financial strength of the entrant relative to that of the incumbent.

## Models of Stochastic Dynamic Demand

Uncertainty can enter the problem in a number of ways. We have already modeled one such situation where the time of entry was made stochastic. We will now examine a more basic level of uncertainty, namely uncertainty in the dynamic demand equation itself. More generally, we will also see what happens when both sources of uncertainty exist, i.e.

when the time of entry as well as the dynamic demand are stochastic. Initially, in the interest of tractability we will restrict ourselves to the simplest monopoly case where advertising is the only strategic variable or control, i.e. the monopoly model of the second section. In the last part of this section we will examine the duopoly version of the stochastic dynamic demand model under uncertainty.

The cumulative sales is modeled as a functional of a Wiener process. We recall that the dynamic demand equation in the deterministic case was $dx/dt = (M-x)a$. A stochastic version of the equation which takes into account random effects in dynamic demand is specified as follows:

$$dx=(M-x)adt+\sigma dW$$

where $\sigma$ is the infinitesimal standard deviation parameter of x. The underlying Wiener process has mean 0 and unit variance. $\sigma$ is a function of time and the cumulative sales x. The infinitesimal variance is the variance of dx which is clearly distinguished from the variance of x itself. The above equation is a stochastic differential equation and is discussed in detail by Malliaris and Brock (1982). Equations of this type can be solved with Ito's lemma, which provides the basic transformation rule of stochastic calculus. A Poisson process can be used in place of Brownian (Wiener) Motion to model the uncertainty here.

The objective function to be maximized is given below:

$$\text{Max } E_{x_0}\left[\int_0^{T_1}[q(M-x)a-\frac{1}{2}a^2]dt\right]$$

where $x(0) = x_0$ is the initial condition, and $E_{x0}$ the conditional expectation operator.

What we now have is a stochastic optimal control problem. The usual method of solving such a problem is Bellman's equation (discussed in Malliaris and Brock (1982)). However this method may not always yield an interpretable solution. Hence we will appeal to an alternative approach developed by Bismut (1975) who used the general methods of convex analysis to analyze stochastic optimal control problems. Bismut's approach consists of the formulation of a stochastic Hamiltonian corresponding to the objective function above, which, for the above problem is:

$$H=q(M-x)a+\lambda(M-x)a+y\sigma$$

Bismut's approach involves the decomposition of the adjoint variable into its mean component and its random component $y$ which represents the instantaneous uncertain changes in demand. Quantitatively, Bismut defined $y$ as the infinitesimal covariance between $\lambda$ and $W$. Tapiero (1978) pointed out that a solution for $y$ is equivalent to an instantaneous optimal attitude towards risk since the expected profit criterion is being used. If the firm is a risk taker $y$ will be positive while it will be negative if it is a risk averter. This approach can be intuitively interpreted as the derivation of the certainty equivalent profits of future uncertain profits where y encompasses the firm's utility of risk taking or averting as the case may be.

The following are the first order conditions of the stochastic Hamiltonian:

$$\frac{\partial H}{\partial a}=0$$

$$d\lambda=-\frac{\partial H}{\partial x}dt+ydW+dN$$

The dN term in the second condition above denotes changes in the prediction of all long term uncertainties which are not captured in the dW term. In terms of our model this would mean that firms obtain additional information over time regarding the effect of such uncertainties. As in Tapiero (1978) we assume that this is not the case, and hence set dN = 0. Using this simplification we write the specific first order conditions for our problem:

$$\begin{gathered} a=(q+\lambda)(M-x) \\ d\lambda=(q+\lambda)adt+ydW \end{gathered} \tag{4.3}$$

Differentiating both sides of the first equation of (4.3) above, and substituting appropriately we get (detailed derivation provided in the appendix):

$$\begin{gathered} da=[y(M-x)-\sigma(q+\lambda)]dW-\sigma ydt \\ =>E(da)=-\sigma ydt \end{gathered} \tag{4.4}$$

Following the usual practice in working with such models, we do not compute dW/dt. dW/dt does not exist as a physically realizable process but does so as a generalized stochastic process which can be given a precise meaning in the theory of generalized functions. However, dW/dt provides a convenient idealization of white noise in continuous time. Note that these subtleties disappear once expectations are taken, because by taking expectations of (4.4) we can write:

$$E(da) = -\sigma y dt \Rightarrow \frac{d(E(a))}{dt} = -\sigma y$$

since the expectation and differentiation operators commute. Equation (4.4) above allows us to state the following proposition:

Proposition 4.3: The optimal advertising of the monopolist with stochastic dynamic demand will decrease on average if the firm is a risk taker and increase on average if the firm is a risk averter.

Proof: $E(da) < 0$ if $y > 0$ and $E(da) > 0$ if $y < 0$. The proposition follows from Bismut's interpretation of $y$ stated earlier.

The intuition behind proposition 4.3 is that the risk taking monopolist decreases advertising because there is scope for additional profits with lower advertising costs which rise faster than advertising effectiveness. In contrast, the risk averting firm will choose to hedge the uncertain cumulative sales with increasing advertising effectiveness. It may be recalled that we obtained the result that advertising will be kept constant in the deterministic case. The stochastic nature of the dynamic demand in the present model implies that the monopolist has to adopt some kind of attitude towards risk is relevant in its decision making. The altered behavior of the firm is reflected in the way it utilizes advertising. When this additional factor of risk attitude interacts with the considerations present in the deterministic case, a constant advertising policy is no longer optimal.

The risk averting monopolist advertises more so as to guard against the variability of sales. We can show that with an increase in advertising the variance of dx will go down. Putting $y = M-x$ we can write the equation of motion as follows:

$$dy=-aydt+\sigma dW$$

where $dy = -dx$.
With this transformation we now have a Ornstein-Uhlenbeck process (Arnold 1974). The Ornstein-Uhlenbeck process has well defined first and second order moments which can be used directly here to obtain a expression for the variance of cumulative sales. The variance of the cumulative sales process can be written as,

$$V(x)=\frac{1-e^{-2at}}{2a}$$

Differentiating V(x) w.r.t. a we get:

$$\frac{\partial V}{\partial a}=\frac{1}{2a^2}[e^{-2at}(1+2at)-1]$$

We know that for an exponential function,

$$e^x=1+x+\frac{x^2}{2}!\ldots\ldots>1+x$$

which implies that $e^{2at} > 1 + 2at$. Using this inequality in the expression for the partial derivative of the variance w.r.t a, we get the result that increased advertising decreases variance of cumulative sales.

If we consider the myopic monopoly situation when the uncertain time of entry the results change. In terms of the stochastic current value Hamiltonian (obtained as before using Merton's theorem), the first order conditions are:

$$\frac{\partial H_c}{\partial a}=0=>a=(q+\lambda)(M-x)$$
$$d\lambda^c=-\frac{\partial H^c}{\partial x}dt+ydW+dN+r\lambda^c=>d\lambda_c=[(q+\lambda)a+r\lambda_c]dt+ydW$$

where $H^c$ = current value stochastic Hamiltonian

$\lambda^c = e^{rt}\lambda$ = current value multiplier.

Using these first order conditions we derive da in the same way as in the previous model of this section:

$$\begin{aligned}da&=[y(m-x)-\sigma(q+\lambda_c)]dW+[r(M-x)\lambda_c-\sigma y]dt\\&=>E(da)=[r(M-x)\lambda_c-\sigma y]dt\end{aligned}\tag{4.5}$$

Equation (4.5) is the expression for the change in advertising in this case where uncertainty prevails at two levels. Unlike the first situation discussed in this section, the expected value of the change in advertising no longer depends only on the risk attitude because of the presence of the second source of uncertainty. It will be equal to the expected value of change in advertising obtained previously only if r (the time of entry distribution parameter) is zero, in which case the second model collapses to the first one. Due to the presence of the dual levels of uncertainty the risk attitude no longer unambiguously determines whether the expected change in advertising will be positive or negative. The uncertainty regarding the possible time of entry is now reflected in the discount rate which affects the expected change in advertising. The discount rate has a direct and an indirect effect through the current value marginal valuation.

Proposition 4.4: The optimal advertising of a risk taking myopic monopolist will increase on average if the current value marginal valuation of his cumulative sales is negative or zero.

Proof: In (4.5) $E(da) < 0$ if $y > 0$ and $\lambda_c < 0$ since $r$, $(M-x) > 0$. The risk taking monopolist will decrease his advertising unambiguously as in the previous model only if the current value marginal valuation is negative. The expected value of the marginal valuation of cumulative sales

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should be negative given fixed population and no repeat purchase. The risk averting monopolist's optimal advertising could increase or decrease over time depending on the magnitude of the rate of discount and the size of the unpenetrated market. A high value of the discount rate will result in higher advertising to begin with since the risk averter will want more sales quickly while a low value will decrease advertising over time as the penetration increases over time.

We will now briefly examine the consequences of extending the model to the competitive case. The stochastic dynamic demand equation extended to the duopoly case is given below:

$$dx_i=(m-x)[a_i+\gamma(a_i-a_j)]+\sigma dW \qquad (4.6)$$

where $i \neq j$ and $i = 1,2$.

It is evident that we are subjecting the cumulative sales of both duopolists to the equivalent random disturbance terms. This is a reasonable assumption given the fact that there is no inherent difference in the products offered by the two firms. The stochastic Hamiltonian for the differential game and the first order conditions are given below:

$$H_i=(q_i+\lambda_i)(m-x)[a_i+\gamma(a_i-a_j)+\sigma dW]-\frac{1}{2}a_i^2+\lambda_i(m-x)[a_j+\gamma(a_j-a_i)+\sigma dW]$$

$$\frac{\partial H_i}{\partial a_i}=0 => a_i=(m-x)[q_i(1+\gamma)+\lambda_i]$$

$$d\lambda_i=[q_i(a_i+\gamma(a_i-a_j))+\lambda_i(a_i+a_j)]dt+ydW$$

Differentiating both sides of the above first order condition and substituting appropriately we get (detailed derivation given in the appendix):

$$E(da_i)=[-q_i(m-x)a_j(1+2\gamma)-2\sigma y]dt \tag{4.7}$$

Proposition 4.5: A risk taking duopolist will decrease its advertising.
Proof: The first term in the expression above will always be negative while the second term will be negative as long as the duopolist is a risk taker since y will then be positive.

The risk taking duopolist decreases its advertising on average since the leftover market diminishes over time in same way as it did for the risk taking monopolist (proposition 4.3). Competition does not affect the direction of the movement of advertising over time since competitive advertising has an uncertain effect given the stochastic nature of dynamic demand. The risk averting duopolist will determine the direction in which to change its advertising based on the variance of cumulative sales. A high variance will induce it to increase its advertising on average while a low variance will decrease it on average.

The models of this chapter demonstrate how uncertainty can affect the optimal behavior of monopolists and duopolists in a dynamic setting. We have restricted ourselves to the case of a new consumer durable. Uncertainty can enter the decision problem of the manager in a number of ways. Here we have discussed the effect of an uncertain time of entry and stochastic dynamic demand. We found that under uncertain time of entry, the myopic monopolist finds it optimal to decrease advertising over time. When a monopolist chooses to erect an entry barrier using advertising it is optimal for it to reduce price over time. With stochastic dynamic demand we find that the firm's attitude towards risk is an important factor for the determination of the expected time paths of its marketing variables. In the next chapter we will conduct an empirical investigation of both static and dynamic noncooperative behavior in a duopoly setting.

# CHAPTER 5

# ESTIMATION OF ADVERTISING EFFECTS

## Introduction

In the last two chapters we have theoretically examined dynamic game theoretic models of non-cooperative behavior with and without stochasticity. While insights derived from the competitive are useful it is necessary to empirically analyze the nature of firms' noncooperative behavior. In fact as Weitz (1985) remarks, the value of this approach depends on the useful empirical research that it spawns. In this chapter we will examine empirically the existence of different types of non-cooperative behavior where firms use advertising and/or prices as competitive weapons. We will adopt the approach suggested by Hanssens et al (1990), who remark that the joint estimation of reaction functions and sales response functions will lead to a more accurate specification of the sales response function. The reaction functions vary according to the behavioral assumption made about the prevailing type of competitive behavior.

Lambin, Naert and Bultez (1975) generalized the reaction function concept in an important way. In previous studies dealing with competitive reaction like Naert (1971), competitors were assumed to react with the same marketing instrument as the one used by a specific firm. This implies that a price decrease will be necessarily be met by a price reduction on the part of the competition. Lambin et al call this the simple competitive case and remark that the more realistic and appropriate case is a model of multiple competitive reaction, given the marketing mix concept. A competitor may react to a change in price by changing price, advertising, quality and promotion or any combination thereof.

Hanssens (1980) extended the work of Lambin et al to a time series framework. The model made an important distinction between market expansive and competitive effects of the marketing mix variables and explored the relation between competitive reactions and effectiveness of marketing variables. Hanssens argued that principles of time series analysis can be used to gain insight into the structure of the market and competitive decision making. He points out that the importance of time series analysis is inversely related to the prior knowledge about the market, which is not very much since data for the competition's market actions is usually not available. In his econometric investigation of the US domestic air travel market, Hanssens found that the flight scheduling has a market expansive or a competitive effect depending on the competitor. Advertising was not found to have a direct effect on market share.

Slade (1986) has used the conjectural variations approach to estimate non-cooperative behavior. Past efforts in that direction include Roberts (1984) and Spiller and Favaro (1984). The approach consists of using the conjectural variations parameter to estimate different types of oligopolistic behavior. The behavior modes tested by Slade include price-taking, price-matching, cartel behavior and cartel with a price taking fringe. The estimation was carried out using primary data gathered from thirteen gas stations in Vancouver. All the market structure hypotheses mentioned above were rejected by the analysis though systematic relationships were found between firm and station characteristics for all variables. For example, ownership and degree of vertical integration are found to affect both conjectures and Lerner concentration indices.

Chu (1989) modeled advertising competition using the conjectural variations approach and estimated conjectural variations in advertising. Data is obtained pertaining to a frequently purchased consumer packaged goods industry and used to estimate sales response models for the two brands using the seemingly unrelated regression technique. The estimated parameters are then used to derive estimates of conjectural variation in advertising for both firms. Chu concludes that the two firms in his sample have consistent beliefs about each others' behavior, and are not likely to engage in a costly advertising war. The consistency of the beliefs allows the prediction of the competitors' future advertising spending.

In this chapter the conjectural variations approach will be used to test for different types of market behavior. Dynamic behavior will be tested using the Cournot type of reaction function where present period prices and advertising depend on the previous period's choices. The data used in this study is for two firms only and hence unlike in Slade (1986) does not require the construction of competitors' price index. In keeping with the idea of a marketing mix, we consider advertising and price as determinants of the sales response instead of restricting ourselves to advertising alone as in Chu (1989).

We will also investigate asymmetries in market response due to the presence of advertising thresholds. Asymmetric market response is an area that has received relatively little attention in the literature. This is surprising in view of the fact that many consumer phenomena are characterized by threshold effects, and existence of thresholds is one reason for asymmetries in functional relationships. The well known S-shaped curve of advertising response is linked to a threshold effect (Little 1979, Lilien and Kotler 1983). The initial small slope of the S-shaped function was said to occur due to advertising threshold effects. Raman and Bass (1988) modeled price threshold effects in reference price theory. Other work on asymmetric response includes Bemmaor (1984) and Aykac, Corstjens and Gautschi (1984). Simon's ADPULS model (1982) involves a special asymmetry to describe jumps in sales levels due to increases in advertising, and is based in the psychological theory of Adaptation Levels. This work extends Simon's model and places it in a more general context, which makes it possible to examine the general problem of asymmetric response.

In the second section of this chapter we discuss the theoretical basis of the models to be estimated. The econometric model used is developed in the third section where the data used here is also discussed briefly. The results of the tests of oligopolistic behavior are analyzed and interpreted in the fourth section. In the concluding section the threshold effect of advertising is discussed and estimated.

## Models of Oligopolistic Behavior

Bowley (1924) was the first to propose the idea of conjectural variations when he postulated that a firm believes that the price it chooses affects the price choice of its rivals and this should be taken into account when choosing its optimal price. The behavioral assumption that the firm makes about how its price affects that of its competitor is called the conjectural variation. Friedman (1983) captures the Bowley model in the following first order conditions where the profit, $\pi_i$ of firm i depends on its own price and firm j's (the competitor's) price:

$$\begin{aligned}\pi_1^1(p_1,p_2)+\pi_2^1\phi'(p_1)=0\\ \pi_2^2(p_1,p_2)+\pi_1^2\phi'(p_2)=0\end{aligned}$$

where $\pi_j{}^i = \partial\pi^i/\partial p_j$ and $\phi_i{}'(p_i) = p_j$ represents what firm i conjectures how firm j's price will change. The conditions above will be the same for different conjectural variations assumptions on the part of the firms. The Bertrand equilibrium[1] is the simplest to characterize since it implies that the competitor firm does not change its current price. This translates into zero conjectural variation, i.e. $\phi_1{}' = \phi_2{}' = 0$. The Stackelberg leader follower model implies that one of the firms, the follower, acts as a passive Cournot player changing its prices in response to the initiatives of the second firm, the leader. This equilibrium requires sequential actions where the leader announces its price first which is then followed by the follower. The conjectural variation for the follower will be zero while that of the leader will be non-zero, the actual value depending on the demand and cost functions of the two firms.

The conjectural variations approach has been criticized as inconsistent because firms have a one period horizon in an essentially multiperiod model. Firms believe that past behavior will be repeated over and over again in every period. Since the firms can observe the past choices of each other they can easily see when the decisions are not repeated. Friedman (1983) suggests using $p_{jt} = \phi_i(p_{i,t-1})$ to represent what firm i thinks firm j will do. If firm j does not behave according to what firm i expects then it will revise its $\phi_i$ to conform to the changed behavior on the part of firm j. The reaction function is based on the observation that firms are aware of the activities and presence of competitors and this alters its behavior in subsequent planning periods. When the current action of the firm depends on previous period actions the reaction function is Cournot in nature.

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[1] The Cournot equilibrium was originally formulated in terms of quantities instead of prices. The Cournot conjecture in terms of prices was originally called the Bertrand conjecture. Friedman (1986) refers to the Cournot conjecture in prices as the Cournot equilibrium.

Equilibrium in the case of the reaction function specified above occurs when the firm maximizes its discounted profits given the reaction functions of the rival firms in each period. Friedman (1983) points out that the natural way to tackle such a problem is with the use of dynamic programming where the Tth period maximization problem is solved under the condition that the T-1th period is governed by the solution calculated for that period for all T. The firm's best strategy can be derived as sequence of T distinct reaction functions, each one specific to a particular remaining time horizon. Such a sequence is optimal because in each period the price is optimal given all prices (own and competitor's) in the previous period. When the horizon tends to infinity Friedman (1977) shows that the reaction functions converge to a single one, thus giving one long-run reaction function which is a function of the previous period prices. All the first derivatives of this reaction function are expected to be positive. The intuition behind this is that if every firm charges a higher price (within some limits) all firms tend to gain. Thus any one firm should be willing to charge higher prices the higher the prices being chosen by the others.

The equilibrium in terms of reaction functions consists of reaction functions for the firms such that profits cannot be increased by changing them. Each firm maximizes its discounted profits given the reaction function of the others. Friedman (1983) remarks that the existence of this equilibrium has not been proved though its impossibility has not been demonstrated either. Existence of an equilibrium can be shown under the convergence assumption that a price increase of a dollar yesterday leads to a less than a dollar's increase in today's price. If this were not the case prices could theoretically become infinitely high leading to infinite profits. Under this assumption the vector of prices of all the firms will converge to some steady state value.

The reaction function approach outlined above has two major advantages over the conjectural variations approach discussed earlier. The reaction function provides a link between choices and outcomes of different periods. The prices chosen in one period influence the profits gained in the subsequent period. Dynamic behavior is captured in this framework as opposed to the conjectural variations approach. The conjectural variations approach implies simultaneous choice in a multiperiod setup and cannot be called truly dynamic since time does not appear explicitly anywhere. The second advantage of the reaction functions is that it does away with the consistency problem encountered with conjectural variations. Under conjectural variations firms do not revise their expectations of the competitors's behavior even when they observe that these expectations are not being met.

## Econometric Model and Estimation Issues

The model on which the econometric estimation of static competitive reaction is based is a two equation system for each firm - a sales response function and a first order profit maximizing condition. This model is similar to that of Slade (1986) but has been modified to accommodate advertising expenditure as well as prices. The sales response function proposed here was used by Gazmi and Vuong (1988) and is given by:

$$q_{it}=\alpha_{i1}+b_{i1}p_{it}+b_{i2}p_{jt}+c_{i1}a_{it}+c_{i2}a_{jt} \qquad (5.1)$$

$$i \neq j, \ i,j = 1,2$$

where

$q_{it}$ = the quantity sold by firm i at time t

$p_{it}$ = firm i's real price at time t and

$a_{it}$ = square root of firm real advertising expenditure at time t.

The price and advertising expenditure can be deflated using the consumer price index. The firm's profit maximization problem is stated as:

$$\text{Max } (P_{it}-mc_i)q_{it}-a_{it} \qquad (5.2)$$

where $mc_i$ = constant marginal cost of firm i.

The first order conditions obtained by differentiating (5.2) after substituting (5.1) with respect to price and advertising are the following:

$$\begin{aligned} q_{it}+(p_{it}-mc_i)(b_{i1}+b_{i2}\theta_i^p)=0 \\ (p_{it}-mc_i)(c_{i1}+c_{i2}\theta_i^a)-2a_{it}=0 \end{aligned} \qquad (5.3)$$

where $\theta_i^p$ = firm i's conjecture about rival's response to a price change and

$\theta_i^a$ = firm i's conjecture about rival's response to a change in advertising expenditure.

We have formulated the conjectural variations model to provide a benchmark for the performance of the dynamic Cournot (in prices) model. When the stochastic specifications of (5.2) and (5.3) is given the equations can be estimated as a system of simultaneous equation for i=1,2.

The conjectures can be used to test for different types of economic behavior of the firms. The Bertrand behavioral hypothesis can be tested by seeing if the conjectural variations for price and advertising are both zero. Alternative firm behavior is exact price and advertising matching behavior where rivals exactly match price decreases and advertising increases, i.e. the conjectures in prices and advertising are equal to one. The hypothesis of the Stackelberg type of leader follower behavior can be tested by testing to see if the leader's conjecture is nonzero while that of the follower is zero.

The dynamic competitive behavior of firms can be estimated using Cournot type reaction functions discussed in the previous section. The competitive reaction has an obvious impact on the sales response function. Accounting for such reaction will enable firms to obtain more accurate estimates of the parameters of the sales response functions. With this in mind it seems to be logical to carry out estimation of reaction functions simultaneously with sales response function as has been suggested by Hanssens et al (1990). If firms have a correctly specified reaction function then they can make better assumptions about competitor behavior for decision making purposes.

In keeping with our earlier discussion the correct dynamic specification of the reaction function is current period price and advertising as a function of previous period own and rival prices and advertising respectively, i.e.:

$$a_{it}=\alpha_{i3}+e_{i2}a_{i,t-1}+e_{i3}a_{j,t-1},\quad i\neq j,\ i,j=1,2 \qquad (5.4)$$

$$p_{it}=\alpha_{i2}+d_{i2}p_{i,t-1}+d_{i3}p_{j,t-1},\quad i\neq j,\ i,j=1,2 \qquad (5.5)$$

These reaction functions to be estimated are the limiting reaction function which are the profit maximizing strategy of the firm i if it has an infinite time horizon. The Cournot assumption made here is that the rival firm j will not move away from its profit maximizing reaction function.

The following is the general model which is set up in order to test the hypothesis suggested in the discussion above:

$$q_{i1}=\alpha_{i1}+b_{ii}p_{it}+b_{ij}p_{jt}+c_{ii}a_{it}+c_{ij}a_{jt}+popgr \qquad (5.6)$$

$$p_{i1}=\alpha_{i2}+d_{i1}q_{it}+d_{i2}p_{i,t-1}+d_{i3}p_{j,t-1} \qquad (5.7)$$

$$\begin{aligned} a_{i1}=\alpha_{i3}+e_{i1}p_{it}+e_{i2}a_{i,t-1}+e_{i3}a_{j,t-1} \text{ if } a_{it}>0 \\ \text{else } a_{2t}=0 \end{aligned} \qquad (5.8)$$

where i,j = 1,2 and i≠j.

Equation (5.6) is the linear sales response function stated in (5.1) earlier. The difference between (5.1) and (5.6) lies in the 'popgr' term which is a trend variable taking into account the factor that population grows over time and leads to trend growth in sales revenue over time. Equations (5.7) is derived from the first order condition involving price contained in (5.3). A new coefficient is substituted for the quantity term (which is defined by the coefficients in (5.6)). The lagged own price and rival's price terms have been added to accommodate the dynamic model which suggests that current price and advertising depends on lagged values of own and rival price and advertising respectively. The coefficients of the lagged terms are zero for the static models while the coefficients for the other terms are set to zero for the dynamic formulation. Equation (5.8) is essentially similar to (5.7) except that the first order condition involves price instead of quantity.

Since the system of equations 5.6-5.8 is simultaneous in nature we need to check the identifiability of the system before proceeding with estimation to ensure that unique estimates of the coefficients of the equations are obtained. The order condition which is a necessary condition for identification is satisfied for all the six equations. When we check the necessary and sufficient condition which is the rank condition we find that equations 5.6 are not identified. In order to ensure identifiability these equation are modified to include promotional expenditure and expenditure on store displays for each firm's sales response equation. The final form of equation (5.6) is as follows:

$$q_{i1}=\alpha_{i1}+b_{ii}p_{it}+b_{ij}p_{jt}+c_{ii}a_{it}+c_{ij}a_{jt}+sd_i+lp_i+popgr \qquad (5.6)$$

where $lp_i$ = promotional expenditure on firm i's product and

$sd_i$ = expenditure on store displays for firm i's product.

Local promotional and store display are appropriate variables to include in the sales response functions since such expenditure is aimed at creating awareness at the retail outlets for the products.

The data set on which we will test our hypothesis about non-cooperative behavior consists of a sales level, price and advertising expenditure data for a household consumer product on a bi-monthly basis between 1952 and 1966. Due to the confidential nature of the data, brands and the exact product cannot be revealed. The product is such that the composition is identical across both brands. The price of this product is low in terms of the total household budget. The market has two main brands which dominates in terms of market share where firm 1 has 13.5% of the market and firm 2 has 50% of the market on average. The total market share of the private labels in the market exceed 10% but data by label is not available. The rest of the market is made up of smaller brands for which only partial data is available. This information on the average market shares suggests a duopolistic market structure with possible leadership by firm 2. A look at the period by period market share for the two main firms reveals that firm 1 is losing market share over time while firm 2 is gaining market share over time. The price data was provided for three different sizes. However since data was missing for the smallest and the biggest size in a number of periods the medium size price was used as the price for the product. The nominal advertising expenditure was deflated by the consumer price index in order to account for the inflation component of the advertising expenditure each period. The monthly population growth was calculated using estimates published in Hollmann (1990).

When estimating advertising reaction functions the possibility of zero reaction by a firm must be accounted for. In some periods a firm may not respond at all to the advertising campaign of its competitor. This possibility is obviously relevant only in the case of advertising since prices presumably never drop to zero for either firm. When a dependent variable in a system (in this case advertising), has a number of observations clustering around a particular value it is called a censored variable. This requires that the estimation be carried out using the Tobit model which involves maximum likelihood estimation taking into account that the residuals of the censored dependent variable equation has a censored normal distribution. Maddala (1983) provides a detailed discussion of this type of model and the appropriate estimation procedures. We did not adopt the Tobit model here because in the case of firm 2 there was a positive level of advertising in every period while firm 1 had close to zero levels of advertising in only five time periods. In such a scenario the gains from a Tobit model estimation are likely to be insignificant.

The estimation procedure we adopted was a seemingly unrelated regression estimation of the entire system. The seemingly unrelated regression estimation procedure is recommended when the disturbance terms of the equations in the system are thought to be correlated. The disturbance terms in the model proposed could easily be correlated since the two firms are expected to retaliate to each other's move in terms of the strategic variables price and advertising. The general model here is a linear one and hence can be estimated without any convergence problems. However, the restrictions implied by static Bertrand and matching behavior imply cross-equation restrictions which turn the system into a nonlinear simultaneous one. The dynamic behavior implies only exclusion restrictions and hence keeps the system linear. The nonlinear seemingly unrelated regression procedure is used to estimate the static Bertrand and matching cases. The estimation was carried out using the SAS statistical software system.

## Tests for Oligopolistic Behavior

Hypothesis testing for three different kinds of oligopolistic behavior was carried out using the model detailed in equations (5.6) - (5.8). Bertrand behavior in both the static and dynamic sense as well as matching behavior was tested for. In all cases the behavior was tested for price and advertising separately and with the two taken together. The expressions for the price and advertising conjectural variations can be obtained by solving the firm's profit maximization problem formulated in terms of the sales response function (Slade 1986) and a fixed unit cost. The conjectural variations terms are written in terms of the parameters of the model and can be used to test different theories of oligopolistic behavior by setting them to different values. These expressions are shown below and we note that they take the value 0 for Bertrand behavior and the value 1 for the matching case.

$$\theta_i^p = \frac{dp_j}{dp_i} \Rightarrow -\frac{(1+d_{i1}b_{i1})}{d_{i1}}$$

$$\theta_i^a = \frac{da_j}{da_i} \Rightarrow \frac{2e_{i1}-c_{i1}}{c_{i2}}$$

where $i = 1,2$ and $i \neq j$. The following set of exclusion restrictions applies in both cases:

$$d_{i2}=d_{i3}=e_{i2}=e_{i3}=0$$

The dynamic case can be tested with exclusions restrictions on (5.7) and (5.8) since current period price and advertising is said to be determined solely on the basis of own lagged values. The restrictions can be stated as,
for price and advertising variables.

$$d_{i1}=0$$
$$e_{i1}=0$$

The likelihood ratio test suggested in Gallant and Jorgenson (1979) was used to test the hypothesis of oligopolistic behavior with the unrestricted model, i.e. equations (5.6) - (5.8) without any restrictions. The test statistic follows a chi square distribution with the number of restricted parameters determining the degrees of freedom. In all cases the alternative hypothesis of various types mentioned above were rejected at both 95% and 99% levels of confidence. No statistically significant evidence of nonzero conjectural variations was found which implies that a leadership model with firm 2 as the leader can also be rejected. The lack of strategic interaction is may be explained by the fact that the product in question is very cheap and hence firms probably feel that probably the consumer will buy whatever he or she sees first on the supermarket shelves.

Inspite of the rejection of all the theories of oligopolistic theories it is interesting to note that dynamic behavior in advertising alone came the closest to not being rejected and was closely followed by dynamic Cournot behavior in price alone and with advertising. The only other case which is performs at reasonable level is the static Cournot behavior in advertising case. These results indicate that advertising competitive behavior is the likeliest form of strategic behavior that occurred in the market. The dynamic behavior hypothesis appears to perform better than the static Bertrand hypothesis. However, since none of the hypothesis were accepted these conclusions about relative performance should be treated with caution.

The unrestricted model's superior performance calls for a closer examination of the individual equations in the system. Tables 5.1-5.4 below shows the parameter estimates with respective t ratios within parenthesis. The R squares of the respective equations are shown in the last column.

Table 5.1 - SUR Parameter Estimates for Sales Response Functions
(Endogenous Variables)

| | $p_1$ | $p_2$ | $a_1$ | $a_2$ |
|---|---|---|---|---|
| $q_1$ | 13.73<br>(0.67) | -1.86<br>(-0.07) | 11.95<br>(2.41) | 6.29<br>(1.42) |
| $q_2$ | 68.55<br>(5.22) | 18.73<br>(0.4) | 23.61<br>(1.87) | 16.93<br>(1.49) |

Table 5.2 - SUR Parameter Estimates for Sales Response Functions
(Exogenous Variables)

| | $sd_i$ | $lp_i$ | popgr | R-Sq. |
|---|---|---|---|---|
| $q_1$ | 1.45<br>(0.29) | 0.93<br>(1.62) | 43.5<br>(4.42) | 0.67 |
| $q_2$ | 2.08<br>(1.63) | 2.19<br>(1.63) | 48.95<br>(16.55) | 0.94 |

The estimates presented in Tables 5.1 and 5.2 shows a good fit for firm 2. A positive own price coefficient for both firms is seen to be insignificant in terms of the t ratios. As mentioned before, the product in question is a cheap household consumer nondurable. In such cases price is likely to be an unimportant factor in the buying decision. However, the rival price coefficients of the sales response function are significant for firm 2 which implies that price increases by firm 1 helped raise sales of firm 2 significantly. The reverse does not appear to hold for firm 1. This asymmetry could result from firm 2 advertising in a manner which highlights higher prices being charged by firm 1. The estimated parameters are positive for own as well as rival advertising expenditure. The information aspect of advertising suggests a possible explanation for the positive magnitude of the coefficients. Primary demand for the product went up with advertising of both firms rather than serving to persuade consumers to switch brands. In fact, if either firm realized it could free ride on the other's advertising. The reason why the firms do not do that is they are unable to separate the persuasion effect of advertising leading to switching between brands and the effect advertising has on primary demand. Furthermore, while a firm may have an estimate of what a rival firm is currently spending on advertising, it has no idea as to what will happen in the future. If the rival firm builds up sufficient goodwill it may reduce advertising to generate greater profits and thus indirectly reduce the sales growth of a firm.

Store displays and local promotions are less significant for firm 1 as opposed to firm 2. Both these determinants of sales are quite likely to be subject to some sort of threshold effect. Firm 2, the larger corporation producing a number of household products, possibly having better distribution facilities, is expected to be in a much better position to utilize store display and local promotions to its advantage. Population growth has the expected positive effect on sales since the expansion of primary target market implies more sales for a household consumption good. The coefficient of population growth indicates the rate at which primary demand for the product was expanding.

Table 5.3 - SUR Parameter Estimates for Advertising Equations

| | $p_i$ | $a_1(-1)$ | $a_2(-1)$ | R-Sq |
|---|---|---|---|---|
| $a_1$ | 0.82<br>(1.32) | 0.26<br>(2.56) | 0.008<br>(0.11) | 0.59 |
| $a_2$ | -2.21<br>(-3.72) | -0.02<br>(-0.19) | 0.51<br>(5.87) | 0.33 |

Note: The (-1) denotes a one period lag.

Table 5.4 - SUR Parameter Estimates for Price Equations

| | $q_i$ | $p_1(-1)$ | $p_2(-1)$ | R-Sq |
|---|---|---|---|---|
| $p_1$ | 0.00007<br>(1.33) | 0.54<br>(6.14) | 0.09<br>(1.6) | 0.09 |
| $p_2$ | 0.00002<br>(1.36) | 0.14<br>(-0.19) | 0.28<br>(2.96) | 0.35 |

Note: The (-1) denotes a one period lag.

The R squares for the price and advertising equations shown in Tables 5.3 and 5.4 above reflect the fact that neither firm was following any significantly consistent advertising or pricing strategy with respect to each other. The pricing policy of each firm appears to be only significantly affected by own price of the previous period while the same is true for advertising. These results indicate that possibly simple rules like percentage of last period advertising and fixed prices were followed by the firms. We note that the price equation has negligible explanatory power for firm 1 while both price and advertising show small explanatory power for firm 2. Given such behavior one can argue for the hypothesis that firm 1 is a follower. However, as stated earlier, there is no statistically significant evidence of nonzero conjectures by either firm. This leads us to conclude that some mild follower behavior is being exhibited by firm 1 with firm 2 as the leader.

While it is acceptable that price would not have a significant effect on sales for a cheap product it is somewhat surprising to find that current period advertising too is not significant. One explanation is that the lagged advertising is what drives sales here rather than current period advertising. This explanation is consistent with the goodwill view of advertising which suggests that past advertising contributes to a stock of goodwill which has a significant effect on sales. One way of examining the effect of lagged variables on sales volume is by using the Koyck model where a lagged value of the dependent variable is used to capture the effects of infinitely lagged independent variable terms. The details of the derivation of such models is discussed in detail in Maddala (1977). Applying the model to our case we get the following specification of the sales response functions:

$$q_{i,t}=\alpha_i+\beta_{i1}q_{i,t-1}+\beta_{i2}a_{i,t-1}$$

where i = 1,2.

Table 5.5 - OLS Parameter Estimates for Lagged Sales Response Functions

| | $q_i$(-1) | $a_1$(-1) | Int. | R-Sq |
|---|---|---|---|---|
| q1 | 0.915<br>(16.8) | 3.00<br>(1.83) | 2.88<br>(0.7) | 0.89 |
| $q_2$ | 1.008<br>(30.4) | 5.68<br>(3.57) | -29.9<br>(-0.9) | 0.95 |

Note: The (-1) denotes a one period lag.

Table 5.5 above shows the results of OLS estimation of the lagged specification of the sales response functions. The intercepts are insignificant as is expected when first differences are used in a stochastic specification. Both the lagged sales term and one period lagged advertising are significant. This implies that the main impact on sales for both firms comes from goodwill accumulation from past period advertising. The lagged sales term encompasses advertising effects from all previous periods, This term can be directly interpreted as the customer holdover effect under which last period's customers continue to buy in the current period. The firms continue to invest in advertising because of the existence of this dynamic effect of advertising. This dynamic effect was not revealed when before since we had only the current period advertising of the firms regressed on sales.

### Threshold Effects in Advertising

The empirical analysis of the previous section reveals a positive effect of advertising on sales without taking into account threshold effects of advertising. Such effects are important in markets where advertising is the main competitive weapon. Adaptation level theory suggests that the current advertising level alone cannot satisfactorily explain sales response. The relation of current advertising to past advertising levels must be considered; this requires distinguishing between an absolute stimulus level (current advertising) and a stimulus. Simon (1972) hypothesizes that the effect of these differential increments is to reach more people more frequently or more intensively. Adaptation level theory implies that these differential stimuli would induce an initial response which levels off gradually but not completely.

Simon's ADPULS model is theoretically based on the concepts discussed above. It is predicated on the notion that a valid advertising model should include a response component due to the effect of stimulus level and a response component due to the (asymmetric) effect of a stimulus differential. These two effects are assumed to be additive. The stimulus differential is represented by the deviation of current advertising ($A_t$) from the adaptation level. The adaptation level is operationalized by the preceding period's advertising ($A_{t-1}$). Thus, the stimulus differential is operationalized by the variable $\Delta A_t = A_t - A_{t-1}$. The general version of Simon's model is

$$q_t = f(A_t, \bar{A}_t, q_{t-1}) + \max\{o, g(\Delta A_t)\} \qquad (5.9)$$

where

$q_t$ = sales in period t

$A_t$ = advertising in period t

$\bar{A}t$ = competitive advertising in period t

g() is a positive, monotonic transformation of $\Delta A_t$. Simon chooses the simplest possible specification of (5.9). His estimated model is

$$q_t = a + \lambda q_{t-1} + \beta \log A_t + \gamma \max\{o, \Delta A_t\}$$

The lagged sales term is interpreted as customer holdover (Kotler 1971). This interpretation has been widely used and discussed in the literature (Clarke 1976, Naert and Leeflang 1978, Parsons and Schultz 1976). The term $\beta \log A_t$ allows for diminishing marginal returns to advertising.

We assume that the sales response function is one of two functions, depending on whether $\Delta A_t \leq A_o$ or $\Delta A_t \leq A_o$, where $A_o$ is an unknown parameter to be estimated. Thus, we specify

$$q_t = a_1 + \lambda_1 q_{t-1} + \beta_1 \log A_t + \gamma \Delta A_t \qquad (5.10)$$
$$\text{if } \Delta A_t > A_o$$

$$q_t = a_2 + \lambda_2 q_{t-1} + \beta_2 \log A_t \qquad (5.11)$$
$$\text{if } \Delta A_t \le A_o$$

Simon's model is nested in ours; it is obtained as a particular case when $A_o = 0$, $a_1 = a_2$, $\lambda_1 = \lambda_2$, and $\beta_1 = \beta_2$. In the specification, we allow for the possibility that $A_o \neq 0$. A priori, we expect $A_o \geq 0$. Moreover, we allow for the possibility that $a_1 \neq a_2$, $\lambda_1 \neq \lambda_2$, and $\beta_1 \neq \beta_2$. This enables us to examine possible asymmetric effects; in particular $\lambda$ and $\beta$ may vary depending on whether $\Delta A_t > A_o$ or $\Delta A_t \leq A_o$. This system generalizes Simon's specification in several ways. Firstly, it allows the threshold $A_o$ to be some positive quantity and not necessarily zero. Secondly, it allows for different marginal impacts of advertising during periods of advertising increases and periods of advertising decreases ($\beta_1 \neq \beta_2$). Furthermore, the lagged sales term, generally interpreted as a measure of brand loyalty, may also vary between periods of advertising increases and decreases ($\lambda_1 \neq \lambda_2$). The well known fact that most marketing managers are reluctant to cut back on advertising expenditures (Little 1979) implies that most managers would expect $\lambda_1 \geq \lambda_2$. We will investigate the existence of a positive threshold level here. In Simon's model (1982), sales are assumed to exhibit an upward jump whenever there is an advertising increment. Thus, the threshold is implicitly taken to be zero, i. e. if $\Delta A_t > 0$, there is an upward jump in sales. We hypothesize an upward jump in sales only if $\Delta A_t > A_o$, where $A_o \geq 0$. In other words, a minimum threshold must be exceeded by $\Delta A_t$ $(= A_t - A_{t-1})$ in order to induce an upward jump in sales.

The estimation problem consists of estimating the parameters $\alpha_1$, $\alpha_2$, $\lambda_1$, $\lambda_2$, $\beta_1$, $\beta_2$, $\gamma$ and $A_0$. The system of equations (5.10) and (5.11) are referred to as switching regressions or a two phase regression system. The estimation procedure consists of dividing observations into two sets of data based on changes in advertising expenditure. All non-negative changes in advertising are used to find the maximum level of the likelihood function and the threshold level is found by finding the level of change in advertising which maximizes the likelihood function of the two equation system. The switching regressions methodology was suggested in Hinkley (1971) and Goldfeld and Quandt (1973).

Table 5.6 below shows the maximum likelihood at various threshold levels of change in advertising for firm 2. The analysis was not performed on data for firm 1 because of the relative lack of variation in the advertising expenditure. The table shows the value of the likelihood function at some selected levels of advertising increments. We can see that the maximum occurs at the 3.96 level which is a positive threshold level. This threshold level is not a very large one which is not surprising given that firm 1 had a much lower advertising budget in general and also did not advertising levels very much. The threshold is likely to be higher in product markets where there are a large number of firms since the increment has to account for competitive advertising by rival firms.

Table 5.6 - Likelihood Values for Various Threshold Levels

| Threshold Level of Advertising Increments | Likelihood Function Value |
|---|---|
| 0 | -442.918 |
| 1.98 | -443.121 |
| 2.62 | -442.354 |
| 3.96 | -442.224 |
| 4.81 | -444.143 |
| 5.99 | -457.554 |
| 7.96 | -459.740 |
| 9.87 | -483.058 |

Note: Advertising expenditure increments are in thousands of real dollars.

Tables 5.7 below shows the maximum likelihood parameter estimates with their respective t ratios for the two regressions with the threshold level at 3.96 and the regression where the entire data set is used to regress sales on lagged sales and advertising. The third regression was carried in order to perform the likelihood ratio test on the significance of the two phase regressions versus a single regression equation. This test evaluates whether the maximized likelihood value significantly differ between the two cases. The test showed that the hypothesis of no

threshold effect was rejected at the 99% level of confidence.

Table 5.7 - Maximum Likelihood Parameter Estimates of Two Phase and Single Equation of the Sales Response Function.

| Equation | $\lambda_i$ | $\beta_i$ | $\gamma$ |
|---|---|---|---|
| (5.10) | 0.98<br>(49.51) | 3.20<br>(3.15) | 7.70<br>(2.88) |
| (5.11) | 1.09<br>(30.12) | 0.58<br>(0.47) | (Does not apply) |
| (5.11) (with full data set) | 0.96<br>(55.74) | 1.84<br>(2.55) | (Does not apply) |

The parameter estimates shown in Table 5.7 show that advertising clearly has a far more significant effect when the increment in advertising is above the threshold level. In fact advertising is not significant when the threshold level is not exceeded. The advertising and lagged sales parameters are significant when the entire data set is used but the two phase regression provides a better fit as mentioned earlier. Our estimation results indicate support for the hypothesis that advertising effects are more pronounced when the level of advertising expenditure is incremented by more than a threshold level of change.

# CHAPTER 6
# CONCLUSIONS

In the previous chapters we have examined the nature of dynamic optimal advertising policies of firms in both monopoly and duopoly environments. In a deterministic setting we obtained the result that the monopolist will find it optimal to advertise at a constant rate over a finite horizon. This implies that new product managers can afford to hold advertising at a constant level during the period of time when there is no threat of competition. The firms can do this because constant price cost margins along with no discounting is assumed. These assumptions are reasonable when we are dealing with a short time horizon. In longer time horizons prices of new products are likely to fall and profits should be discounted to obtain present value of future profits. Advertising cost will be discounted in such time horizons which should only shift the dynamic path of advertising downward.

The optimal constant level of advertising varies directly with the square root of the price cost margin. This is a reflection of the fact that the profit margin on products often drive the amount of advertising done by a firm. The constant price cost margin drives this result. The role of advertising in this situation is to obtain greater penetration which implies greater profits rising linearly with market penetration. During the introductory period for a durable product the price can be held at a high level and advertising maintained at a level which can be supported by the price.

It is interesting to note that once we relax the assumption that advertising is the sole control variable, the result that a monopolist should hold advertising constant does not change. With the introduction of price as a control variable the optimal advertising policy for the monopolist is again a constant one. The constant advertising allows us to derive the optimal price at each point in time which is the same as the one derived by Eliashberg and Jeuland (1986) for the case when price was the sole control variable. This implies that the product manager need only concern himself with setting price during the time period since advertising is constant.

The optimal level of advertising is shown to be inversely related to the price sensitivity parameter when both price and advertising are control variables. The managerial implication of this result is that when price cutting does not increase revenues a monopolist should advertise more to achieve greater market penetration. The presence of a competitor does not alter this result since a rival's advertising does not affect directly affect the price sensitivity parameter of a firm. A nonlinear specification of the sales response function (nonlinear in price) also gives the same result thus demonstrating the robust nature of the proposition for which empirical support has been found.

In the competitive situation where there are two firms in the market, the nature of the optimal advertising policy for the duopolist is shown to be a declining rate of advertising. The rival firm's advertising has a negative effect on the advertising effectiveness of the firm. The sales response function used has the advertising differential between firms driving sales. Thus a duopolistic firm can increase profits by decreasing advertising over time as long as it advertises at ahigher rate than its competitor.

The environment for most managers is an uncertain one where they are seldom certain about the effectiveness of their advertising campaigns or when the market will be invaded by another firm. Analyzing situations like these produces results which are substantially different from the ones derived for the deterministic case. Restricting our attention to the case of a myopic monopolist, we find that it is optimal for a myopic monopolist facing uncertain time of entry to decrease advertising as long as the discounted marginal valuation of cumulative sales are negative. Unlike in the deterministic case, a constant level of advertising is no longer optimal. When the number of units sold decreases over time the advertising intensity should decline to maximize profits.

A monopolist may attempt to deter entry by advertising at a high rate so as to give the impression that any entrant will have to match such advertising. Even though such behavior may or may not deter entry, the pricing policy that will accompany such strategic behavior will be that of declining prices over time as greater market penetration is achieved by the monopolist. However, the success of such a strategy may depend on the financial ability of the incumbent firm to sustain short term losses. A potential entrant may enter on the basis that such behavior on the part of the incumbent will not be optimal after entry has occurred. In such a case an advertising and/or a price war can ensue depending on the attitude of the incumbent. If the advertising is maintained at pre-entry levels, profits will fall for the incumbent in the short run as long as the entrant refuses to be scared off.

The risk attitude of the manager often determines what sort of business strategy is followed. By incorporating uncertainty into the sales response function we were able to analyze the effect of the risk attitude of the managers. We found that the optimal advertising of a monopolist will decrease (increase) on average if the firm is a risk taker (averter). The risk taking monopolist expects a positive random effect while the risk averting monopolist expects a negative one. These expectations translate into decreasing and increasing advertising respectively. In the case of a duopolist we find that the result holds for the risk taker. The risk averting duopolist on the other hand chooses to decrease or increase advertising depending on a high or low cumulative sales variance. Another result tells us that a risk taking myopic monopolist (one who chooses to maximize only current period profits) will increase his or her advertising on average if sales are increasing at a decreasing rate. The risk taking manager chooses to advertise aggressively in a falling sales situation because stochastic sales response can potentially produce higher than expected sales. The myopic attitude of the monopolist drives his or her desire to maximize current horizon profits since entry can occur at an uncertain point in time in the future.

In the empirical investigation we have concentrated on analyzing noncooperative behavior in both a static and a dynamic context. The product chosen was a nondurable mature household product, which was dominated by two firms during the period for which data is available. We tested matching and Bertrand behavioral conjectures in a static framework for both advertising and price variables. One limitation of this approach is that noncooperative behavior is essentially dynamic in nature and accordingly a dynamic model is formulated where current period advertising and price depends on previous period advertising and price decisions respectively of both firms. The sales response function can be better formulated by managers keeping in mind the dynamic nature of competitive reaction and the possibility that in some periods there may be no reaction from competitors. Historical price and advertising estimates for competitors are available to firms on occasions and should be used in order to obtain more reliable sales forecasts.

The empirical results obtained indicate that price rivalry is not an important characteristic in the market investigated. In such a market where the price of the product is low relative to the household budget advertising is the more important competitive weapon. Lagged advertising was found to be an important determinant of sales indicating that some accumulation of goodwill due to past period advertising was in effect. The examination of the threshold effect of advertising indicated that it is important for managers to realize that sales are affected by advertising only when advertising is increased by a certain specific amount which will possibly vary with the nature of the product. It is important to have an estimate of the threshold level or else advertising will not be as effective as expected. It should be noted that the sales response function is driven by a different set of parameters when the firm chooses to increase advertising by less than the threshold amount.

The emphasis of the theoretical and empirical work in the previous chapters has been on the optimal strategies in a dynamic environment. The solutions that were obtained were open-loop Nash equilibrium strategies for differential games. One of the major areas for future research lies in obtaining closed-loop or feedback Nash solutions for models similar to those discussed here. Such strategies would be more realistic for firms who often revise their pricing and advertising decisions based on new information accumulated through the planning period. Derivation of feedback Nash strategies would ensure that such information is incorporated in the firm's optimal strategies.

Entry in the models discussed has been restricted to one in order to allow the incorporation of the uncertainty of the time of entry. Future research is necessary to examine the occurrence and effect of multiple uncertain entries into a market. Such a model would suggest the process of evolution of a market for a new product from a monopoly to an oligopoly. One of the questions that arise in such a situation is what kind of entry deterring strategy will be adopted by the firms. In a multifirm market the possibility exists that one or a group of incumbents may choose to free ride on the entry deterring activities of the others. The dynamic modeling of such a process should yield interesting insights about the durability of entry deterring activities of firms.

The empirical investigation can be modified to accommodate interaction terms within the sales response functions estimated. The theoretical analysis indicated that such interaction can exist between the price and advertising variables. An accurate specification of the sales response function would include such interaction terms. A competitive advertising threshold can be estimated by considering the differential advertising levels between firms. It appears to be reasonable that firms should have to advertise more than its competitors to be effective. A threshold level of the difference in advertising could be estimated. An index of competitive advertising would be necessary in markets with more than two firms. The availability of accurate data on other elements of the marketing plan would also allow improvement of the specification of the sales response function.

For such empirical models to be of use to working managers it must be incorporated into a decision support system which allows managerial judgement to be used along with information from historical data. An appropriate means of doing this is the construction of an expert system based on the expert knowledge of the manager. Essentially this methodology entails the construction of a set of if-then rules which embody the knowledge of the manager. Construction of the type of decision support systems would be an important steps towards converting theoretical models of firm behavior into practical rules which can be used by managers for optimal economic decision making.

# APPENDIX
# DETAILED MATHEMATICAL DERIVATIONS

## Derivation of (3.4)

$$a_1 = \frac{q_1}{T_1}\left(MT_1 - \int_0^{T_1} x_1 dt\right)$$

$$\frac{dx_1}{dt} = (M - x_1)\, a_1$$

Using the standard form of the differential equation of the first order $\frac{dy}{dx} + Py = Q$ (P and Q are constants and the integrating factor $e^{\int P dx}$ we have

$$x_1 = e^{-\int a_1 dt}\left\{\int e^{\int a_1 dt} Ma_1\, dt + c\right\}$$

$$= e^{-a_1 t}\left\{Ma_1 \frac{1}{a_1} e^{a_1 t} + c\right\}$$

$$= M + \frac{c}{e^{a_1 t}}$$

where c = constant of integration.

Using the initial condition, $x_1(0)=0$, we get $c = -M$ which implies

$$x_1 = M(1 - 1/e^{a_1 t})$$

$$\int_0^{T_1} x_1\,dt = M\int_0^{T} (1 - \frac{1}{e^{a_1 t}})\,dt$$

$$= M\left[t + \frac{e^{-a_1 t}}{a_1}\right]_0^{T_1}$$

$$= MT_1 - \frac{x_1(T_1)}{a_1}$$

## Proof of Proposition 3.2

Differentiating the advertising first order condition with respect to t, and rearranging the adjoint equation we can write the following

$$\dot{a}_i = [m-x]\dot{\lambda}_i + [q_i(1+\gamma)+\lambda_i](-\dot{x}_i - \dot{x}_j) \qquad \text{(A-1)}$$

$$\dot{\lambda}_i - \lambda_i(a_i + a_j) = q_i[a_i + \gamma(a_i - a_j)] \qquad \text{(A-2)}$$

Substituting (3.2) in (A-1) and rearranging terms we get

$$\dot{a}_i = [m-x][\dot{\lambda}_i - \lambda_i(a_i + a_j) - q_i(1+\gamma)(a_i + a_j)] \qquad \text{(A-3)}$$

Substituting (A-2) in (A-3) we get

$$a_i = [m - x]\ \{q_i[a_i + \gamma(a_i - a_j)] - q_i(1 + \gamma)(a_i + a_j)\}$$

$$= -\ [m - x]\ q_i(2\gamma + 1)\ a_j$$

## Proof of Proposition 3.3

Differentiating (3.9) with respect to t on both sides we get

$$\dot{a}_1 = (1/k - 2p_1 - kp_1^2)(-\dot{x}_1) + (M - x_1)(-2 + 2kp_1)\dot{p}_1$$

Substituting (3.8) and (3.10) we get

$$\begin{aligned} a_1 &= -(M - x_1)(1/k - 2p_1 + Kp_1^2)\,a_1(1 - kp_1) \\ &\quad - 2(M - x_1)(1 - Kp_1)\left(-\frac{1}{2}\right)(1/k - P_1)(1 - Kp_1)\,a_1 \\ &= 0 \end{aligned}$$

## Proofs of Proposition 3.8 and 3.9

The adjoint condition for the Hamiltonian can be written as

$$(P_1 - c_1 + \lambda_1) = -\frac{P_1}{\eta} \qquad \text{(A-4)}$$

Substituting (A-4) in the advertising first order condition we get

$$a_1 = -\frac{(M - x_1)\,p_1^{\eta+1}}{\eta} \qquad \text{(A-5)}$$

Taking log of both sides of (A-5) we get

$$\text{Log } a_1 = -[\text{Log }(M - x_1) + (\eta + 1)\text{ Log } p_1 - \text{Log } \eta] \qquad \text{(A-6)}$$

Differentiating (A-6) implicitly with respect to time and yields

$$\frac{1}{a_1}\,\dot{a}_1 = -\left[\frac{1}{(M - x_1)}(-\dot{x}_1) + (\eta + 1)\,\frac{1}{p_1}\,\dot{p}_1\right] \qquad \text{(A-7)}$$

where $\dot{x}_1 = (M - x_1)\, a_1 p_1^{\eta}$.

Differentiating the price first-order condition with respect to time and substituting the adjoint equation gives

$$\begin{aligned}\dot{P} &= -\frac{\eta}{1+\eta}\,\dot{\lambda}_1 \\ &= -\frac{\eta}{1+\eta}\,(p_1 - c_1 + \lambda_1)\, a_1 p_1^{\eta} \\ &= -\frac{\eta}{1+\eta}\left(-\frac{p_1}{\eta}\right) a_1 p_1^{\eta} \\ &= \frac{a_1}{1+\eta}\, p^{\eta+1} \qquad \text{(A-8)}\end{aligned}$$

Substituting $\dot{x}_1$ and (A-8) in (A-7) we get

$$\dot{a}_1 = a_1\left[-a_1 p_1^{\eta} + a_1 p_1^{\eta}\right] = 0$$

Differentiating (A-6) partially with respect to $\eta$ we get

$$\frac{1}{a_1}\frac{\partial a_1}{\partial \eta} = \frac{1}{\eta} - \operatorname{Log} p_1$$

$$\Rightarrow \frac{\partial a_1}{\partial \eta} = a_1\left(\frac{1}{\eta} - \operatorname{Log} p_1\right) < 0$$

given $a_1$, $p_1$ are positive while $\eta$ is negative.

## Proof of Extension of Merton's Theorem

For a Poisson process N(t) we may write

$$P\{N(t+\epsilon) - N(\epsilon) = n\} = e^{-m(t)}\frac{m(t)^n}{n!} \qquad \text{(A-9)}$$

where $m(t) = \int_0^t r(s)ds$ for all s, $t \geq 0$.

What (A-9) implies is that the number of events in any interval of length t has a Poisson distribution with mean m(t).

Let $T_1$ = Time of entry,

then $N(t) = \begin{cases} 1 \text{ if entry occurs.} \\ 0 \text{ if entry does not occur.} \end{cases}$

The probability that entry did not occur in the interval t can be written as follows

$$P[T_1 > t] = P[N(t) = 0]$$

$$= 1 \cdot e^{-m(t)} = e^{-m(t)}$$

Given the above we can write

$$P[T_1 < t] = [1 - e^{-m(t)}] = F(t)$$

where F(t) is the cumulative distribution function (cdf) of entry occurring. The corresponding probability distribution function (pdf) can be written as

$$f(t) = \frac{\partial (F(t))}{\partial t} = m'(t) e^{-m(t)}$$

$$= r(t) e^{-\int_0^t \lambda(s) ds}$$

since $m'(t) = \frac{\partial}{\partial t} \left[ \int_0^t r(s) ds \right] = r(t)$.

Following Merton's (1971) method of proof we can now write,

$$E_0\left[\int_0^{T_1} U(C_1, t)\,dt\right] = \int_0^{\infty} r(T_1)\, e^{-\int_0^{T_1} r(v)\,dv}\left[E_1 \int_0^{T_1} U(C_1, t)\,dt\right]dT_1$$

$$= \int_0^{\infty}\int_0^{T_1} r(T_1)\, e^{-\int_0^{T_1} \lambda(v)\,dv}\, g(t)\,dt\,dT_1$$

where $g(t) = E_1\,[U(C_1, t)]$

$E_0$ = conditional expectation operator over all random variables including $T_1$,

$E_1$ = conditional expectation operator over all random variables excluding $T_1$

The expression above can be rewritten as

$$\int_0^{\infty}\left\{\int_0^{T_1} g(t)\,dt\,d\left[-e^{-\int_0^{T_1} r(v)\,dv}\right]\right\}dT_1$$

Integrating by parts,

$$\left[e^{-\int_0^{T_1} r(v)\,dv}\int_0^{T_1} g(t)\,dt\right]_0^{\infty} + \int_0^{\infty} e^{-\int_0^{T_1} r(v)\,dv}\, g(T_1)\,dT_1$$

The first term above equals zero since,

$$\int_{0}^{0} = 0 \text{ and } e^{-\infty} = 0$$

Thus,

$$E_{0}\left[\int_{0}^{T_{1}} U(C_{1}, t)dt\right] = E_{1}\int_{0}^{\infty} e^{-\int_{0}^{\infty} r(v)dv} U(C_{1}, t)dt$$

(substituting $g(t) = E_{1}[U(c, t))$

Merton's theorem is thus shown to be true when r, the parameter of the exponential distribution of $T_1$, is a function of time.

## Derivation of Equation (4.4)

Differentiating both sides of (4-3) we get

$$\begin{aligned} da &= (q + \lambda)(-dx) + (M - x)d\lambda - \mathrm{Cov}(ydW, \sigma dW) \qquad \text{(A-9)} \\ &= (q + \lambda)[-(M - x)adt - \sigma dW] + (M - x)[(q + \lambda)adt + ydW] - \sigma ydt \\ &= -(q + \lambda)(M - x)adt + (q + \lambda)(M - x)adt \\ &\quad - \sigma(q + \lambda)dW + y(M - x)dW - \sigma ydt \\ &= [y(M - x) - \sigma(q + \lambda)]dW - \sigma ydt \end{aligned}$$

## Derivation of Equation (4-7)

We obtain an equation similar to (A-9), by differentiating the first order condition for advertising in the competitive case

$$da_i = [q_i(1+\gamma)+\lambda_i](-dx)+(m-x)d\lambda_i - \mathrm{Cov}(ydW, 2\sigma dW) \qquad \text{(A-10)}$$

$$dx = dx_i + dx_j = (m-x)q_i(a_i+a_j)+2\sigma dW \qquad \text{(A-11)}$$

(A-11) was obtained by using the dynamic demand specified in (4-6). Using (A-11) and the adjoint equation, (A-10) can be rewritten as

$$da_i = (m-x)\{[q_i[q_i+\gamma(a_i-a_j)] + \lambda_i(a_i+a_j)]d_t + ydW\}$$
$$- [q_i(1+\gamma)+\lambda_i]\{(m-x)(a_i+a_j)dt + 2\sigma dW\} - 2\sigma ydt$$

Simplifying and collecting terms we get

$$da_i = q_i(m-x)a_j(1+2\gamma)dt - 2\sigma ydt + \{(m-x)\ y - 2\sigma[q_i(1+$$
$$\gamma) + x_i]\}\ dW$$

Taking expectations of both sides we get

$$E[da_i] = -\ q_i(m-x)q_i(1+2\gamma)dt - 2\sigma ydt$$

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